International Journal of Mathematical Archive-6(2), 2015, 209-223
Available online through www.ijma.info ISSN 2229 – 5046

MULTI-ITEM, MULTI-CRITERIA AND MULTI-CONSTRAINT LEVEL ECONOMIC PRODUCTION PLANNING INVENTORY PROBLEM: A GEOMETRIC PROGRAMMING APPROACH

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(Received On: 24-07-15; Revised & Accepted On: 28-02-15)

ABSTRACT

In this paper, we have formulated a multi-item, multi-objective economic production and marketing planning inventory problem under the constraints of space capacity and the total allowable shortage cost. Here the multi-objective functions of the problem are combined to a single objective function by global criterion method. Geometric Programming technique based on global criterion method is used to find the optimal solutions for different preferences on objective functions. Finally, the solution procedure is illustrated by a numerical example.

Keywords: *Global Criterion method; hybrid method; demand; set-up cost; Multi-objective; inventory.*

1. INTRODUCTION

In most of the inventory problems, the unit price of an item is considered as independent in nature. Actually, it relates to the demand of that item. When the demand of an item is high, it is produced in large numbers. Fixed costs of production are spread over a large number of items. Hence the unit cost of the item decreases i.e. the unit price of an item inversely relates to the demand of that item. Cheng [5] formulated the EOQ problem with this idea and solved through GP method. Duffin, Peterson and Zener [6] discussed the basic theories on GP with engineering application in their books. Another famous book on GP and its application appeared in 1976 [3]. They described GP with positive or zero degree of difficulty.

But there may be some problems on GP with negative degree of difficulty. Sinha *et al.* [14] proposed it theoretically. Using Geometric Programming technique Abou-El-Ata and Kotb [1] studied the multi-item EOQ inventory model with varying holding cost under two restrictions and solved by geometric programming technique. Sahidul *et al.* ([12], [13]) discussed modified geometric programming problem and also developed a inventory model solved by modified geometric programming technique.

Similarly the marketing cost which includes the advertisement and promotion cost directly affect the demand of an item. The manufacturing companies increase the advertisement cost and give some advantages (like promotion, incentives) to their sales representatives according to their performances. Lee and Kim [10] studied the marketing planning problem considering such idea to solve by GP method.

The Society becomes more complex and as the competitive environment develops, businesspersons are finding that they require deeming multiple objectives. Almost every imperative real world problem involves more than one objective. In such cases, decision makers evaluate best possible approximate solution alternatives according to multiple criteria. Over the last two decades tremendous amount of research effort has been expanded on multi-objective decision making leading to the publication of many interesting results in the literature ([4], [7], [11]).

The multiple criteria and multiple constraint level linear programming (MC^2LP) problem was first defined and solved by Seiford and Yu [15] and Yu [17]. It is the extension of traditional linear programming problem. The traditional linear programming problem is involved with only single criteria (objective) and a single constraint (resource availability) level. Generally, only single decision maker (DM) for example, the production manager in charge of decision-making designs inventory problems. It has limitations in dealing with real world problems, which involve multiple conflicting criteria. Such problem cannot solve by either linear programming or multi criteria (MC) linear programming. Extending a single DM to a multiple DM setting introduces the multiple resource availability level frameworks to modeling group decision-making problems. The preference values of each involved manager on

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resource availability can be treated as an individual level of the availability. Then, the modified model has multiple discrete levels of resource availability. It is clear that the multiple decision makers may have different preferences in terms of their backgrounds and positions in their firm that results in inevitable conflicts. In addition to the tradeoff between multiple criteria, the 'optimal solution' must reflect the compromise or tradeoffs among the multiple discrete levels of resource availability.

GP method is an effective method to solve a non-linear programming problem. It has certain advantages over the other optimization methods. The advantage here is that is usually much simpler to work with the dual than the primal. This method is now widely used to solve the optimization problem in inventories. Worral and Hall [16] analysed the inventory models with some constraints and solved by GP technique. Hariri and Abou-el-ata [8] and Abou-el-ata and Kotb [1] used GP to solve multi-item inventory problems. Ata, Fergany and Wakeel [2], Jung and Klain [9] developed single item inventory problems and solved by GP method.

In this paper, we have formulated a multi-item, multi-objective economic production and marketing planning inventory problem under the constraints of space capacity and the total allowable shortage cost. Here the multi-objective functions of the problem are combined to a single objective function by global criterion method. Geometric Programming technique based on global criterion method is used to find the optimal solutions for different preferences on objective functions. Finally, the solution procedure is illustrated by a numerical example.

2. MATHEMATICAL MODEL

2.1. Mathematical Formulation:

The following basic notations and assumptions are used in the proposed model.

Notations:

Parameters for *i* (=1,2,…,*n*)-th item are

D_i Demand per unit of time

Q_i Order quantity i.e. number of units ordered per order (decision variable)

Q The decision variable vector of order quantity $(Q_1, Q_2, ..., Q_n)^T$

M_i Marketing expenditure (decision variable),

M The decision variable vector of marketing cost $(M_1, M_2, ..., M_n)^T$

S_i Shortage amount per unit of time (decision variable)

S The decision variable vector of shortage amount, $(S_1, S_2, ..., S_n)^T$

c_{0i} Unit production cost

h_i Inventory holding cost per item per unit time

R_i Amount of stock at time t=0

c_{3i} Set-up cost per cycle

c_3 The set up cost vector $(c_{31}, c_{32}, ..., c_{3n})$

c_{2i} Shortage cost per unit per time

c_2 The shortage cost vector $(c_{21}, c_{22}, ..., c_{2n})$

w_i Storage space area per unit item

w Total available storage area

S^1 Total allowable shortage cost.

Assumptions:

(i) Production is instantaneous with zero lead-time.

(ii) Shortages are allowed.

(iii) The demand rate is directly proportional to the marketing expenditure i.e., $D_i = \alpha_i M_i^{\beta_i}, \alpha_i > 0, \beta_i > 0,$

(iv) The unit production cost is inversely proportional to demand rate i.e., $c_{0i} = \gamma_i D_i^{-\delta_i}$, where $\gamma_i > 0$ is a scalling constant and $\delta_i > 1$ is called price elasticity.

Let the amount of stock is R_i at time $t = 0$. In the interval $(0, T_i = t_{1i} + t_{2i})$, the inventory level gradually decreases to meet demand. By this process the inventory level reaches zero level at time t_{1i} and then shortages are allowed to occur in the interval (t_{1i}, T_i), the cycle then repeats itself. The differential equation for the instantaneous inventory $q_i(t)$ at time t in $(0, T_i)$ is given by

$$\frac{dq_i(t)}{dt} = -D_i \quad for\ 0 \le t \le T_i \tag{2.1.1}$$

with the conditions $q_i(0) = R_i$, $q_i(T_i) = -S_i$ *and* $q_i(t_{1i}) = 0$.

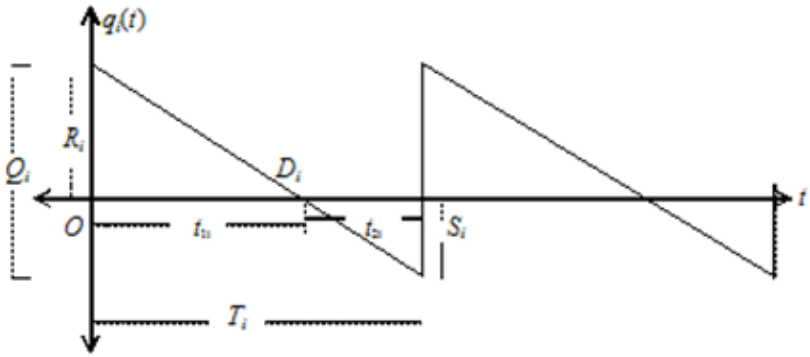

Fig.1: Inventory situation for the i-th item

Solution of the differential equation is given by

$$q_i(t) = R_i - D_i t \qquad for\ \ 0 \le t \le t_{1i}$$
$$= D_i(t_{1i} - t) \quad for\ \ t_{1i} \le t \le T_i$$

So, $D_i t_{1i} = R_i$, $S_i = D_i t_{2i}$, $Q_i = D_i T_i$.

Holding cost = $h_i c_{0i} \int_0^{t_{1i}} q_i(t)dt = \dfrac{h_i c_{0i}(Q_i - S_i)^2}{2Q_i} T_i$.

For each period a fixed amount of shortage is allowed and there is a penalty cost c_{2i} per item of unsatisfied demand per unit time.

Shortage cost = $c_{2i} \int_{t_{1i}}^{T_i} (-q_i(t))\, dt = \dfrac{c_{2i} S_i^2}{2Q_i} T_i$.

Production cost = $c_{0i} Q_i = \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} Q_i$.

Advertisement cost = $M_i Q_i$.

The total inventory related cost = set up cost + holding cost + shortage cost

$$= \sum_{i=1}^{n} \left[h_i c_{0i} \frac{(Q_i - S_i)^2}{2Q_i} T_i + c_{3i} + \frac{c_{2i} S_i^2}{2Q_i} T_i \right].$$

The total average inventory related cost *i*.e.,

$$TC_1(M, Q, S) = \sum_{i=1}^{n} \left[h_i \frac{(Q_i - S_i)^2}{2Q_i} \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} + c_{3i} \frac{\alpha_i M_i^{\beta_i}}{Q_i} + \frac{c_{2i} S_i^2}{2Q_i} \right]$$

$$= \sum_{i=1}^{n} \left[\begin{array}{l} \frac{1}{2} h_i \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} Q_i - h_i \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} S_i \\ + \dfrac{h_i \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} S_i^2}{2Q_i} + c_{3i} \dfrac{\alpha_i M_i^{\beta_i}}{Q_i} + c_{2i} \dfrac{S_i^2}{2Q_i} \end{array} \right] \tag{2.1.2}$$

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And total additional cost= marketing cost + production cost = $\sum_{i=1}^{n}\left[M_i Q_i + c_{0i} Q_i\right]$.

So, total average additional cost i.e., $TC_2(M) = \sum_{i=1}^{n}\left[\alpha_i M_i^{1+\beta_i} + \gamma_i \alpha_i^{1-\delta_i} M_i^{\beta_i(1-\delta_i)}\right]$ (2.1.3)

Note: When shortages are not allowed i.e., when $c_2 \to \infty$ then

$$TC_1'(M,Q) = \sum_{i=1}^{n}\left[\frac{1}{2}h_i\gamma_i\alpha_i^{-\delta_i}M_i^{-\beta_i\delta_i}Q_i + c_{3i}\frac{\alpha_i M_i^{\beta_i}}{Q_i}\right] \quad (2.1.4)$$

$$TC_2'(M) = \sum_{i=1}^{n}\left[\alpha_i M_i^{1+\beta_i} + \gamma_i \alpha_i^{1-\delta_i} M_i^{\beta_i(1-\delta_i)}\right] \quad (2.1.5)$$

The manufacturing organization produces some items and stocks these items in a warehouse. The manufacturing companies or organizations use huge advertisement for their products in order to increase the level of demand. Still they have some limitations regarding total space capacity, total allowable shortage cost etc. In this phenomenon the organization is interested in minimizing the inventory related cost (including set up cost, shortage cost) and additional cost (includes marketing cost and production cost) simultaneously.

So, the problem is to minimize total average inventory costs and also to minimize total average of additional cost under the limitations of space capacity, total allowable shortage cost.

Hence the Multi-Objective Inventory Model (MOIM) is as follows:

$$Min\, TC_1(M,Q,S) = \sum_{i=1}^{n}\left[\begin{array}{l}\frac{1}{2}h_i\gamma_i\alpha_i^{-\delta_i}M_i^{-\beta_i\delta_i}Q_i - h_i\gamma_i\alpha_i^{-\delta_i}M_i^{-\beta_i\delta_i}S_i \\ + \frac{h_i\gamma_i\alpha_i^{-\delta_i}M_i^{-\beta_i\delta_i}S_i^2}{2Q_i} + c_{3i}\frac{\alpha_i M_i^{\beta_i}}{Q_i} + c_{2i}\frac{S_i^2}{2Q_i}\end{array}\right]$$

$$Min\, TC_2(M) = \sum_{i=1}^{n}\left[\alpha_i M_i^{1+\beta_i} + \gamma_i \alpha_i^{1-\delta_i} M_i^{\beta_i(1-\delta_i)}\right]$$

subject to (2.1.6)

$$\sum_{i=1}^{n}\left[w_{1i}(Q_i - S_i)\right] \le w,$$

$$\sum_{i=1}^{n}\left[c_{2i}\frac{S_i^2}{2Q_i}\right] \le S^1,$$

M, Q, S > 0.

The above problem can be written as a Multi-Objective Geometric Programming Problem (MOGPP)

$$Min\, TC_1(M,Q,S) =$$

$$\sum_{i=1}^{n}\left[\frac{1}{2}h_i\gamma_i\alpha_i^{-\delta_i}M_i^{-\beta_i\delta_i}Q_i - h_i\gamma_i\alpha_i^{-\delta_i}M_i^{-\beta_i\delta_i}S_i + \frac{h_i\gamma_i\alpha_i^{-\delta_i}M_i^{-\beta_i\delta_i}S_i^2}{2Q_i} + c_{3i}\frac{\alpha_i M_i^{\beta_i}}{Q_i} + c_{2i}\frac{S_i^2}{2Q_i}\right]$$

$$Min\, TC_2(M) = \sum_{i=1}^{n}\left[\alpha_i M_i^{1+\beta_i} + \gamma_i \alpha_i^{1-\delta_i} M_i^{\beta_i(1-\delta_i)}\right]$$

subject to (2.1.7)

$$\sum_{i=1}^{n}\left[\frac{w_{1i}}{w}(Q_i - S_i)\right] \le 1$$

$$\sum_{i=1}^{n}\left[\frac{c_{2i}S_i^2}{2S^1Q_i}\right] \le 1$$

M, Q, S > 0.

3. MATHEMATICAL ANALYSIS

3.1 Geometric Programming Problem:

The signomial Primal Geometric Programming (PGP) problem be

Min $f(x) = \sum_{i=1}^{n}\sum_{s=1}^{N_0}\sigma_{1s}c_{si}\prod_{k=1}^{N}x_{ki}^{a_{sk}}$ (3.1.1)

subject to $G_j(x) = \sum_{i=1}^{n}\sum_{l=1}^{N_j}\sigma_{2jl}d_{jli}^{j}\prod_{k=1}^{N}x_{ki}^{a_{jlk}} \le 1$

where $\sigma_{1s}, \sigma_{2jl} = +1 \text{ or } -1$

$c_{si}, d_{jli}^{j} > 0,\ x_{ki} \ge 0,\ a_{sk}$ and a_{jlk} are constants

for $i = 1,2,...,n;\ s = 1,2,...,N_0;\ k = 1,2,...,N;\ l = 1,2,...,N_t;\ j = 1,2,...,q.$

It is a PGP problem with Degree of Difficulty (DD) = $nN_0 + n\left(\sum_{j=1}^{K}N_j\right) - (nN+1)$.

The corresponding Dual Programming (DP) problem of (3.1.1) is

Max $dw = \prod_{i=1}^{n}\left[\left(\prod_{s=1}^{N_0}\left(\frac{c_{si}}{w_{si}}\right)^{\sigma_{1s}w_{si}}\right)\left(\prod_{j=1}^{K}\left(\prod_{l=1}^{N_j}\left(\frac{d_{jli}^{j}u_j}{w_{jli}}\right)^{\sigma_{2jl}w_{jli}}\right)\right)\right]$ (3.1.2)

where $u_j = \sum_{i=1}^{n}\sum_{l=1}^{N_j}w_{jli} \qquad j = 1,2,...,q.$

Subject to the normality, orthogonality and non-negativity conditions,

$$\sum_{i=1}^{n}\sum_{s=1}^{N_0}\sigma_{1s}w_{si} = 1,$$

$$\sum_{i=1}^{n}\left[\sum_{s=1}^{N_0}\sigma_{1s}a_{sk}w_{si} + \sum_{j=1}^{K}\sum_{l=1}^{N_t}\sigma_{2jl}a_{jlk}w_{jli}\right] = 0,$$

$$w_{si}, w_{jli} \ge 0,\ i = 1,2,...,n;\ s = 1,2,...,N_0;\ k = 1,2,...,N;\ j = 1,2,...,q..$$

The optimal values of the variables are obtained from the relations

$$c_{si}\prod_{k=1}^{N}x_{ki}^{*a_{si}} = w_{si}^{*}dw^{*},$$

$$d_{jli}^{j}\prod_{k=1}^{N}x_{ki}^{*a_{ji}} = \frac{w_{jli}^{*}}{\sum_{i=1}^{n}\sum_{l=1}^{N_1}w_{jli}^{*}},\quad s = 1,2,..,N_0; i = 1,2,..,n;\ j = 1,2,...,q$$

and optimal value of the objective function is dw^{*}.

3.2 Multi-Objective Geometric Programming Problem (MOGPP):

A multi-objective geometric programming problem can be stated as:

Find $x = (x_1, x_2, ..., x_n)^T$ so as to (3.2.1)

Minimize $f_1(x) = \sum_{i=1}^{T_1^0}c_{1i}^{0}\prod_{r=1}^{n}x_r^{a_{1ir}^{0}}$

Minimize $f_2(x) = \sum_{i=1}^{T_2^0}c_{2i}^{0}\prod_{r=1}^{n}x_r^{a_{2ir}^{0}}$

…….

Minimize $f_k(x) = \sum_{i=1}^{T_k^0} c_{ki}^0 \prod_{r=1}^{n} x_r^{a_{kir}^0}$

Subject to $g_p(x) = \sum_{s=1}^{T_p} c_{ps} \prod_{r=1}^{n} x_r^{a_{psr}} \le 1 \qquad p = 1, 2....., m$

$x>0$,

where $c_{ji}^0(>0), c_{ks}(>0), a_{jir}^0, a_{jir}$ are all real numbers for j=1,2,..,k; i=1,2,.., T_j^0 ; k=1,2,…,m; s=1,2,…, T_k .

Let X be a set of constraints of (3.2.1) such that

$$X = \{x \in \Re^n \mid g_i(x) \le b_j,\ j = 1, 2, ..., m,\ and\ x = (x_1, x_2, ..., x_n)^T\ with\ \ x_i \ge 0\ \ for\ \ i = 1, 2, ..., n\}$$

The multi-objective optimization problem is convex if all the objective functions and the feasible region are convex. Some basic on Pareto optimal solutions are introduced below.

Pareto optimality:

In single objective optimization problems, the main focus is on the decision variable space whereas in the multi-objective framework, we are often more interested in the objective space. In multi-objective programming problems, multiple objectives are usually non-commensurable and cannot be combined into a single objective. In MOGPP, the objectives are simultaneously optimized. But due to an intrinsic conflicting nature among the objectives it is not possible to find a single solution that would be optimal for all the objectives simultaneously. Consequently, the aim in solving MOGPP is to find a compromise or satisfying solution of the DM. There is no natural ordering in the objective space because it is only partially ordered.

However, some of the objective vectors can be extracted for examination. Edgeworth (1987) said that such vectors are those where none of the components can be improved without deterioration to at least one of the other components. This definition is usually called Pareto-optimality, which is laid, by French- Italian economist and sociologist Vilfredo Pareto.

Definition 3.2.1: Let x^* be the optimal solution of the following problem
Minimize $f_r(x) \quad r = 1, 2, ..., k$
subject to $x \in X$.

The point x^* is known as **ideal solution** and r–th objective function value at x^* i.e. $f_r(x^*)$ is known as ideal objective value.

Definition 3.2.2: A solution x^* is said to be a **Complete optimal solution** to the MOGPP (3.2.1) if there exists $x^* \in X$ such that $f_r(x^*) \le f_r(x),\ (r = 1, 2....., k)$ for all $x \in X$.

In general, the objective functions of the MOGPP conflict with each other, a complete optimal solution does not always exist and so Pareto (or non-dominated) optimality concept is introduced.

Definition 3.2.3: A decision vector $x^* \in X$ is **Pareto optimal solution** if there does not exist another decision vector $x \in X$ such that $f_r(x) \le f_r(x^*)$ for all r = 1, 2,……,k and $f_j(x) \ne f_j(x^*)$ for at least one j, j∈{1,2,..,k}.

Definition 3.2.4: A decision vector $x^* \in X$ is **weakly Pareto optimal solution** if there does not exist another decision vector $x \in X$ such that $f_r(x) < f_r(x^*)$ for all r = 1, 2,…,k.

Definition 3.2.5: A solution $x^* \in X$ is said to be a **locally Pareto optimal solution** to the MOGPP if and only if there exists an r < 0 such that x^* is Pareto optimal in $X \bigcap N(x^*, r)$, i.e. there does not exist another $x^* \in X \bigcap N(x^*, r)$ such that $f_i(x) \le f_i(x^*)$.

3.3 Multi Criteria and Multi Constraint Level Non Linear Programming (MC^2NLP)

In general, the Multiple Criteria (or Multiple Objective) Nonlinear Programming (MCNLP) problem takes the following form

$$\text{Min}\left(f_1(x),\ f_2(x),\ldots,\ f_m(x)\right) \tag{3.3.1}$$

subject to $x \in X = \left\{x \in R^n \mid g_j(x) \le b_j,\ j = 1,2,\ldots,k\right\}$

where $m\ (\ge 2)$ objective functions $f_i : R^n \to R$ and $X \subset R^n$.

Note: When m =1 problem (1) reduces to a single objective nonlinear programming problem.

Definition 3.3.1: Let x_r^* be the optimal solution of the following problem

$$\text{Min}\ f_r(x)$$

subject to $x \in X$.

The point x_r^* is known as ideal point and r-th objective function value at x_r^* i.e. $f_r\left(x_r^*\right)$ is called as ideal objective value.

MC^2NLP:

For given an $(k \times p)$ matrix B with columns $B_1,\ldots,B_p$, the convex set generated by B or the convex hull of B denoted by the following set:

$$H(B) = \left\{b : b = \sum_{l=1}^{p} B_{jl}\rho_l,\ j = 1,2,\ldots,k;\ \rho_l \ge 0, l = 1,2,\ldots,p; \text{and} \sum_{l=1}^{p} \rho_l = 1\right\}.$$

Therefore the corresponding MC^2NLP problem is defined in the following form:

$$\text{Min}\left(f_1(x),\ f_2(x),\ldots,\ f_m(x)\right)$$

subject to $x \in X_1 = \left\{x \in R_+^n \mid g_j(x) \le b_j,\ j = 1,2,\ldots,k; \text{for some}\, b \in H(b)\right\}.$

It is easy to see that the *MCNLP* problem is a special case of MC^2NLP problem.

Definition 3.3.2: A pareto optimal solution of problem MC^2NLP with $\rho_l > 0$ is called a potential solution, and a pareto optimal solution of problem MC^2NLP is called a weak potential solution.

Theorem 3.3.1: $x^* \in X_1$ is a weak potential solution of MC^2NLP problem if and only if there is a $\omega = (\omega_1,\ldots,\omega_m) \in R^m, \omega_r \ge 0, \forall r$ such that x^* solves the following problem:

$$\text{Min}\left(\sum_{r=1}^{m} \omega_r \left| f_r(x) - f_r\left(x^*\right)\right|^p\right)^{\frac{1}{p}},\ \text{for } 1 \le p < \infty \tag{3.3.2}$$

subject to $x \in X_1$.

Proof: Since $b \in H(b)$ then there exist $\rho = (\rho_1,\ldots,\rho_p) \ge 0$ with $\sum_{l=1}^{p} \rho_l = 1$ such that $b = \sum_{l=1}^{p} B_{jl}\rho_l$. By applying the scalarization theorem of MCNLP problem, which say that $x^* \in X$ is a pareto optimal solution of the problem MCNLP if and only if there exist $\omega = (\omega_1,\ldots,\omega_m),\ \omega_r > 0, \forall r$ such that x^* solves the problem L_p-problem the result follows immediately. Therefore the proof is complete.

Theorem 3.3.2: (Following Liu, Shi (1994)) For each $\overline{\rho} = \left(\overline{\rho}_1, \overline{\rho}_2,\ldots,\overline{\rho}_p\right) > 0$ with $\sum_{l=1}^{p} \overline{\rho}_l = 1,\ \ x^*$ is a potential solution of MC^2NLP problem:

$$\text{Min}\left(f_1(x),\ f_2(x),\ldots,\ f_m(x)\right) \tag{3.3.3}$$

subject to $x \in X = \left\{x \ge 0 \mid g_j(x) \le b_j,\ j = 1,2,\ldots,k; \text{for some}\, b \in H(B)\right\}$

if and only if x^* is a pareto optimal solution of the MCNLP problem

$$\text{Min}\left(f_1(x), f_2(x), \ldots, f_m(x)\right)$$

subject to $x \in X = \left\{ x \geq 0 \mid g_j(x) \leq \sum_{l=1}^{p} B_{jl}\bar{\rho}_l,\ j = 1,2,\ldots,k \right\}$.

Proof: The result follows from the scalarization theorem of MCNLP problem.

Corollary 3.3.1: x^* is a potential solution of MC^2NLP problem if and only if there exist an $\varepsilon > 0$ and $\omega = (\omega_1, \ldots, \omega_m) \in R^m, \omega_r > 0, \forall r$ such that x^* solves the problem

$$\text{Min}\left(\sum_{r=1}^{n} \omega_r \left| f_r(x) - f_r(x^*) \right|^p \right)^{\frac{1}{p}},\ \text{for } 1 \leq p < \infty \tag{3.3.4}$$

subject to $g_j(x) \leq \sum_{l=1}^{p} B_{jl}\rho_l, \quad j = 1,2,\ldots,k,$

$$\sum_{l=1}^{p} \rho_l = 1, x \geq 0,\ \rho_l \geq \varepsilon > 0,\ \forall l = 1, 2, \ldots, p.$$

3.4 Global Criterion Method:

In this method, the distance between some reference point and the feasible objective region is minimized. The decision maker has to select the reference point and the metric for measuring the distances. In this way, the multiple objective functions are transferred into a single objective function. We suppose that the weighting coefficients w_r are real numbers such that $\omega_r \geq 0, \forall r = 1,2,\ldots,m$ and $\sum_{r=1}^{m} \omega_r = 1$. The weighted L_p - problem for minimizing distances is stated as

$$\text{Minimize } L_p(f(x)) = \left(\sum_{r=1}^{k} \omega_r \mid f_r(x) - f_r(x^*) \mid^p \right)^{\frac{1}{p}} \tag{3.4.1}$$

subject to $x \in X,\ for\ 1 \leq p < \infty.$

Following Chankong and Haimes (1983), the hybrid problem combining L_p and ε - constraint method is as follows

$$\text{Minimize } L_p(f(x)) = \left(\sum_{r=1}^{k} \omega_r \mid f_r(x) - f_r(x^*) \mid^p \right)^{\frac{1}{p}} \tag{3.4.2}$$

subject to $f_r \leq \omega_r,\ x \in X,\ for\ 1 \leq p < \infty,$

where $\omega_r \geq 0,\ \forall r = 1, 2, \ldots\ldots, k; \quad \sum_{r=1}^{k} \omega_r = 1,$

and $\varepsilon_r (\leq f_r(x^*))$ is a real number for all r = 1, 2, ……., k.

For p = 1, Minimize $L_1(f(x)) = \sum_{r=1}^{k} \omega_r \mid f_r(x) - f_r(x^*) \mid$ (3.4.3)

subject to $x \in X$.

The objective function $L_1(f(x))$ is the sum of the weighted deviations, which is to be minimized and the problem (3.4.3) is known as weighted sum problem.

For p = *2*, Minimize $L_2(f(x)) = \left(\sum_{r=1}^{k} \omega_r \mid f_r(x) - f_r(x^*) \mid^2 \right)^{\frac{1}{2}}$ (3.4.4)

subject to $x \in X$.

When p becomes larger, the minimization of the deviation becomes more and more important. The problem (3.4.4) is known as least square problem.

Finally, when $p \to \infty$, the only thing that matters is the weighted relative deviation of one objective function i.e.,

Minimize $L_\infty(f(x)) = \underset{r=1,2,....,k}{Max}\left(\sum_{r=1}^{k} \omega_r \,|\, f_r(x) - f_r(x^*)|\right)$ (3.4.5)

subject to $x \in X$.

Above method is called 'min-max' method or Tchebycheff method. Problem (3.4.5) is non differentiable like its un-weighted counterpart. Correspondingly, it can be solved in a differentiable form as long as the objective and the constraint functions are differentiable and $f_r(x^*)$ is known globally. In this case, the problem (3.4.5) becomes

Minimize λ

subject to $\omega_r\left(f_r(x) - f_r(x^*)\right) \le \lambda$ for all r = 1, 2, ……, k (3.4.6)

$x \in X, \quad \lambda \in R$.

Theorem 3.4.1: The solution of weighted L_p – problem (when $1 \le p < \infty$) is Pareto optimal solution if all the weighting coefficients are positive.

Theorem 3.4.2: The solution of weighted Tchebycheff problem (L_∞) is weakly Pareto optimal if all the weighting coefficients are positive.

Theorem 3.4.3: Weighted Tchebycheff problem has at least one Pareto optimal solution.

Theorem 3.4.4: Let a decision vector $x^* \in X$ be given, solve the problem

Minimize $\sum_{r=1}^{k} f_r(x)$ (3.4.7)

subject to $f_r(x) \le f_r(x^*) \quad for\ all\ \ r = 1, 2, ..., k$

$x \ge 0$.

Let Ø(x^*) be the optimal objective value. The decision vector $x^* \in X$ is Pareto optimal if and only if it is a solution of (3.4.7) so that $\phi(x^*) = \sum_{r=1}^{k} f_r(x^*)$.

Proof: The proofs of the four theorems are followed by Miettinen (1999).

When $f_r(x)$ and (r = 1, 2… k) and $g_j(x)$ (j = 1, 2… k) are posynomial and signomial functions, the problems (3.4.3), (3.4.4) and (3.4.6) may be reduced to a single objective geometric programming problem.

3.5 Global Criterion method to solve the MOGPP (2.1.7):

The Multi-objective Inventory Model (MOIM) may be solved by several techniques. Some of those are Fuzzy geometric programming technique and global criterion method. Here global criterion is used to find the compromise solution of problem (2.1.7). In this method the objective functions are combined to a single objective function.

Let $\omega_r \ge 0,\ r = 1, 2$ are the normalized weights (i.e $\omega_1 + \omega_2 = 1$ corresponding to the objective functions $TC_1(M, Q, S)$ and $TC_2(M)$. TC_1 and TC_2 are the ideal objective values of $TC_1(M, Q, S)$ and $TC_2(M)$ respectively (Deductions are shown in Appendix A). TC_1 and TC_2 are obtained objective functions for $TC_1(M, Q, S)$ and $TC_2(M)$ respectively without taking constraint by using Geometric Programming (GP) method. The weighted L_p-problem according to Miettinen (1999) is as follows

Minimize $U_p(M, Q, S) = \left(\omega_1 \left|TC_1(M, Q, S) - TC_1\right|^p + \omega_2 \left|TC_2(M) - TC_2\right|^p\right)^{\frac{1}{p}}$ (3.5.1)

subject to same constraints as in (2.1.7), for $1 \le p << \infty$.

Case-1: The weighted sum problem i.e. for p = 1 in (3.4.1) is given as

Minimize $U_1(M,Q,S) = \omega_1(TC_1(M,Q,S) - TC_1) + \omega_2(TC_2(M) - TC_2)$

subject to the same constraints as in (2.1.7).

Since ω_1, ω_2, TC_1 *and* TC_2 are independent of the decision variables, so it is enough to solve the following problem

Minimize $V_1(M,Q,S) = \omega_1 TC_1(M,Q,S) + \omega_2 TC_2(M)$ (3.5.2)

subject to the same constraints as in (2.1.7),

where $U_1(M,Q,S) = V_1(M,Q,S) - (\omega_1 TC_1 + \omega_2 TC_2)$.

The above problem (3.5.2) is a signomial Geometric Programming problem with DD = 6n-1 and that can be easily solved by GP method. The optimal solution will be potential solution by section (3.3).

Case-2: The least square problem i.e for p = 2 in (3.5.1) is given as

Minimize $U_2(M,Q,S) = \left[\omega_1(TC_1(M,Q,S) - TC_1)^2 + \omega_2(TC_2(M) - TC_2)^2\right]^{\frac{1}{2}}$ (3.5.3)

subject to the same constraints as in (2.1.7).

To obtain the standard form of GP problem of the above weighted quadratic problem we introduce two new variables y_1 and y_2 which are the upper bounds of $TC_1(M,Q,S) - TC_1$ and $TC_2(M) - TC_2$ respectively $(i.e\ TC_1(M,Q,S) - TC_1 \le y_1\ and\ TC_2(M) - TC_2 \le y_2)$, we may rewrite the above problem (3.5.3) as follows:

Minimize $V_2(M,Q,S,Y) = \omega_1 y_1^2 + \omega_2 y_2^2$ (3.5.4)

subject to $\dfrac{TC_1(M,Q,S)}{TC_1} - \dfrac{y_1}{TC_1} \le 1$

$$\frac{TC_2(M)}{TC_2} - \frac{y_2}{TC_2} \le 1$$

$$\sum_{i=1}^{n}\left[\frac{w_{1i}}{w}(Q_i - S_i)\right] \le 1$$

$$\sum_{i=1}^{n}\left[\frac{c_{2i}S_i^{\,2}}{2S^1 Q_i}\right] \le 1$$

$$M, Q, S, y_1, y_2 > 0$$

where $U_2(M,Q,S) = (V_2(M,Q,S))^{\frac{1}{2}}$.

The problem (3.5.4) is also signomial GP problem with DD =6n+1 and can be easily solved by GP method to obtain the potential solution.

Case-3: The Tchebycheff problem (i.e. for $p \to \infty$) is given as

Minimize $U_3(M,Q,S) = Maximize\left(\omega_1\left|TC_1(M,Q,S) - TC_1\right|, \omega_2\left|TC_2(M) - TC_2\right|\right)$ (3.5.5)

subject to the same constraints as in (2.1.7) .

We introduce a new variable λ, which is maximum value between $\omega_1(TC_1(M,Q,S) - TC_1)$ $and\, \omega_2(TC_2(M) - TC_2)$ $i.e.$ $\omega_1(TC_1(M,Q,S) - TC_1) \le \lambda$ and $\omega_2(TC_2(M) - TC_2) \le \lambda.$

Then the above problem (3.5.5) is reduces to the problem as follows:

Minimize λ (3.5.6)

subject to $\dfrac{TC_1(M,Q,S)}{TC_1} - \dfrac{\lambda}{w_1 TC_1} \le 1$

$$\frac{TC_2(M)}{TC_2} - \frac{\lambda}{w_2 TC_2} \le 1$$

$$\sum_{i=1}^{n}\left[\frac{w_{1i}}{w}(Q_i - S_i)\right] \le 1$$

$$\sum_{i=1}^{n}\left[\frac{c_{2i}S_i^{\,2}}{2S^1 Q_i}\right] \le 1$$

$$M, Q, S > 0.$$

The above problem (3.5.6) is again a signomial GP problem with DD = 8 and can be easily solved by GP technique to obtain the potential solution.

4. NUMERICAL EXAMPLES

A multinational manufacturing company produces different type soft drinks. It is given that the inventory holding cost (h_i) of the drinks are 25% and 22% of the unit production cost, the set-up costs (c_{3i}) are \$130 and \$150 and the shortage costs (c_{2i}) are \$10 and \$15 of two types of drinks respectively. The annual demand (D_i) of the drinks is varies directly with the marketing expenditure i.e., $D_1 = 10M_1^{1.5}$ and $D_2 = 12M_2^{1.8}$ and the unit production cost (c_{0i}) is varies inversely with the demand i.e., $c_{01} = 5.05M_1^{-1.8}$ and $c_{02} = 5.14M_2^{-2.5}$ respectively. The storage area (w_{1i}) per unit item is 3 m^2 and 5 m^2 respectively. The total available storage area and total allowable shortage cost are w = 325 m^2 and S^1 = \$ .096.

The ideal value (as computed in Appendix A) of $TC_1(M,Q,S)$ is TC_1 = \$252.1214 and that of $TC_2(M)$ is TC_2 = \$ 247.4325. The company decides to know the optimal values of the inventory related cost ($TC_1(M, Q, S)$), additional cost ($TC_2(M)$), marketing cost M^*, order quantity Q^*, shortage amount S^*.

The above problem can be formulated as multi-objective geometric programming problem. The potential solutions of this MOGPP by global criterion method for p=1, p=2 and p→∞ are given in Table -1, Table-2 and Table-3 for different preference values (i.e., different weights) of the objective functions.

Table-1: Equal preferences to the two objective functions i.e for (w_1, w_2) = (0.5, 0.5)

p	i	M_i^*	Q_i^*	S_i^*	$TC_1^*(M^*,Q^*,S^*)$	$TC_2^*(M^*)$
1	1	0.83135	41.5412	0.5789	259.5647	
	2	0.85283	39.0147	0.5803		248.8954
2	1	0.82341	39.3608	0.5816	259.2341	
	2	0.84123	38.4219	0.5799		248.9843
∞	1	0.79657	36.0143	0.5657	258.9768	
	2	0.78985	37.6543	0.5549		251.4537

The above Table-1 gives different potential solutions when the DM supplies equal preferences to the inventory related cost objective function $TC_1(M,Q,S)$ and additional cost objective function $TC_2(M)$. Here, $TC_1^*(M^*,Q^*,S^*)$ give minimum value when $p \to \infty$ where as $TC_2^*(M^*)$ give minimum value when p = 1.

Table-2: More preference to the inventory related cost function i.e., (w_1, w_2) = (0.6,0.4)

p	i	M_i^*	Q_i^*	S_i^*	$TC_1^*(M^*,Q^*,S^*)$	$TC_2^*(M^*)$
1	1	0.83547	40.4612	0.5791	259.6534	
	2	0.84674	39.8547	0.5816		249.4537
2	1	0.82164	38.0854	0.5875	259.1432	
	2	0.83135	38.9163	0.5806		249.3152
∞	1	0.79573	36.0143	0.5657	258.9768	
	2	0.78857	37.6543	0.5548		251.4537

The above Table-2 shows different potential solutions when the DM supplies more preference to the inventory related cost objective function $TC_1(M,Q,S)$ than the additional cost objective function $TC_2(M)$. Here, $TC_1^*(M^*,Q^*,S^*)$ give minimum value when $p \to \infty$ whereas $TC_2^*(M^*)$ give minimum value when p = 1.

Table-3: More preference to the additional cost objective function i.e., (w_1,w_2)= (0.4,0.6)

p	i	M_i^*	Q_i^*	S_i^*	$TC_1^*(M^*,Q^*,S^*)$	$TC_2^*(M^*)$
1	1	0.79875	40.3243	0.5612	258.7541	248.5137
	2	0.78443	39.5428	0.5767		
2	1	0.83142	39.9846	0.5875	257.3216	249.1532
	2	0.83522	39.0126	0.5861		
∞	1	0.79706	36.1432	0.5902	257.8757	250.5373
	2	0.78858	37.4376	0.5880		

The above Table-3 shows different potential solutions when the DM supplies more preference to the additional cost objective function $TC_2(M)$ than the inventory related cost objective function $TC_1(M,Q,S)$. Here, $TC_1^*(M^*,Q^*,S^*)$ give minimum value when $p \to \infty$ whereas $TC_2^*(M^*)$ give minimum value when p = 1.

5. CONCLUSION

Here, we have discussed multi-objective geometric programming problem based on global criterion method. We have also formulated the multi-item, multi-criteria inventory optimization model of economic production and marketing planning problem under two constraints. The different objective functions are combined to a single objective function by global criterion method. GP technique is used to derive the potential solutions for different preferences on objective functions. In Table-1, 2 and 3 we have shown the potential solution's of my problem for different preference values of the objective functions. This technique can be applied to solve the different decision making problems (like in inventory and other areas).

ACKNOWLEDGEMENTS

The author wish to acknowledge the "DST PURSE" in the Department of Mathematics, University of Kalyani, Kalyani, Nadia-741235, India for financial assistance.

APPENDIX A:

Working rule for finding the ideal objective values TC_1 and TC_2.

$$Min\ TC_1(M,Q,S) = \sum_{i=1}^{n} \left[\begin{array}{l} \frac{1}{2} h_i \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} Q_i - h_i \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} S_i \\ + \frac{h_i \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} S_i^2}{2Q_i} + c_{3i} \frac{\alpha_i M_i^{\beta_i}}{Q_i} + c_{2i} \frac{S_i^2}{2Q_i} \end{array} \right]$$

subject to M, Q, S >0.

The above problem is a primal geometric programming problem with DD = 2n-1. The corresponding dual problem is

$$Maximize\ v(\rho) = \prod_{i=1}^{n} \left(\frac{h_i \gamma_i \alpha_i^{-\delta_i}}{2\rho_{1i}} \right)^{\rho_{1i}} \left(\frac{h_i \gamma_i \alpha_i^{-\delta_i}}{\rho_{2i}} \right)^{\rho_{2i}} \left(\frac{h_i \gamma_i \alpha_i^{-\delta_i}}{2\rho_{3i}} \right)^{\rho_{3i}} \left(\frac{c_{3i} \alpha_i}{\rho_{4i}} \right)^{\rho_{4i}} \left(\frac{c_{2i}}{2\rho_{5i}} \right)^{\rho_{5i}}$$

subject to the normality and orthogonality conditions

$$\left. \begin{array}{l} \rho_{1i} - \rho_{2i} + \rho_{3i} + \rho_{4i} + \rho_{5i} = 1, \\ -\beta_i \delta_i \rho_{1i} + \beta_i \delta_i \rho_{2i} - \beta_i \delta_i \rho_{3i} + \beta_i \rho_{4i} = 0, \\ \rho_{1i} - \rho_{3i} - \rho_{4i} - \rho_{5i} = 0, \\ -\rho_{2i} + 2\rho_{3i} + 2\rho_{5i} = 0, \\ \rho_1, \rho_{2i}, \rho_{3i}, \rho_{4i}, \rho_{5i} > 0. \end{array} \right\} \quad (i=1, 2, \ldots, 3)$$

Solving the dual variables in terms of ρ_{3i} we get

$$\rho_{1i} = \rho_{3i} + \frac{2\delta_i - 1}{2\delta_i}, \rho_{2i} = 2\rho_{3i} + \frac{\delta_i - 1}{\delta_i}, \rho_{4i} = \frac{1}{2}, \rho_{5i} = \frac{\delta_i - 1}{2\delta_i}. \text{ (i=1, 2,…..,3)}$$

Since the dual variables are always positive so $\delta_i > 1$ (i=1,2,…..,3). Substituting the values of the above dual variables into the dual objective function and then differentiating with respect to ρ_{3i} equal to zero, we get

$$\rho_3^* = \frac{(\delta_i - 1)^2}{2\delta_i} \text{ (i=1, 2,…..,3).}$$

So the values of the dual variables are

$$\rho_1^* = \frac{\delta_i}{2},\ \rho_2^* = \delta_i - 1,\ \rho_3^* = \frac{(\delta_i - 1)^2}{2\delta_i} \rho_4^* = \frac{1}{2},\ \rho_5^* = \frac{\delta_i - 1}{2\delta_i} \quad where\ \delta_i > 1. \text{(i=1,2,…..,3)}$$

Substituting the values of the dual decision variables in the dual objective function of the dual problem we get

$$v^*(\rho^*) = \prod_{i=1}^{n} \left(\frac{h_i \gamma_i \alpha_i^{-\delta_i}}{2\rho_{1i}^*} \right)^{\rho_{1i}^*} \left(\frac{h_i \gamma_i \alpha_i^{-\delta_i}}{\rho_{2i}^*} \right)^{\rho_{21}^*} \left(\frac{h_i \gamma_i \alpha_i^{-\delta_i}}{2\rho_{3i}^*} \right)^{\rho_{31}^*} \left(\frac{c_{3i}\alpha_i}{\rho_{4i}^*} \right)^{\rho_{4i}^*} \left(\frac{c_{2i}}{2\rho_{5i}^*} \right)^{\rho_{5i}^*}.$$

Following Duffin, Peterson and Zener (1967) we get the optimum value of the primal objective function as $TC_1^* = v^*(\rho^*)$.

The decision variables of the primal problem can be obtains from the following primal-dual relationships:

$$\frac{h_i \gamma_i \alpha_i^{-\delta_i} M_i^{-\beta_i \delta_i} Q_i}{2\rho_{1i}^*} = v^*(\rho^*)$$

$$\frac{c_{3i}\alpha_i M_i^{\beta_i}}{Q_i \rho_{4i}^*} = v^*(\rho^*)$$

$$\frac{c_{2i} S_i^2}{2Q_i \rho_{5i}^*} = v^*(\rho^*). \text{ (i=1, 2,…., n).}$$

Solving the above system of non-linear equations, we get

$$M_i^* = \left(\frac{h_i \gamma_i c_{3i}}{2\alpha_i^{\delta_i - 1} \rho_{1i}^* \rho_{4i}^* (v^*(\rho^*))^2} \right)^{\frac{1}{\beta_i(\delta_i - 1)}}$$

$$Q_i^* = \frac{c_{3i}\alpha_i}{\rho_{4i}^* v^*(\rho^*)} \left(\frac{h_i \gamma_i c_{3i}}{2\alpha_i^{\delta_i - 1} \rho_{1i}^* \rho_{4i}^* (v^*(\rho^*))^2} \right)^{\frac{1}{\delta_i - 1}}$$

$$S_i^* = \left(\frac{\rho_{5i}^* c_{3i}\alpha_i}{c_{2i}\rho_{4i}^*} \left(\frac{h_i \gamma_i c_{3i}}{2\alpha_i^{\delta_i - 1} \rho_{1i}^* \rho_{4i}^* (v^*(\rho^*))^2} \right)^{\frac{1}{\delta_i - 1}} \right)^{0.5}. \text{ (i=1, 2,….,n).}$$

The ideal objective value TC_1 is defined as $TC_1 = TC_1^*(M^*, Q^*, S^*)$.

In a similar way, we can find the optimal value of the additional cost objective function TC_2.

Minimize $TC_2(M) = \sum_{i=1}^{n} \left[\alpha_i M_i^{1+\beta_i} + \gamma_i \alpha_i^{1-\delta_i} M_i^{\beta_i(1-\delta_i)} \right]$

subject to M >0.

The above problem is a primal geometric programming problem with DD = n -1. The corresponding dual problem is

Minimize $v(\lambda) = \prod_{i=1}^{n} \left(\frac{\alpha_i}{\lambda_{1i}} \right)^{\lambda_{1i}} \left(\frac{\gamma_i \alpha_i^{1-\delta_i}}{\lambda_{2i}} \right)^{\lambda_{2i}}$

subject to the normality and orthogonality conditions

$\lambda_{1i} + \lambda_{2i} = 1$,

$(\beta_i + 1)\lambda_{1i} + \beta_i (1 - \delta_i)\lambda_{2i} = 0$, (i=1, 2,…., n)

$\lambda_{1i}, \lambda_{2i} > 0.$

Solving the system of linear equations of the above dual problem, we get the dual decision variables are

$\lambda_{1i}^* = \frac{\beta_i(\delta_i - 1)}{\beta_i \delta_i + 1}, \lambda_{2i}^* = \frac{\beta_i + 1}{\beta_i \delta_i + 1}, \quad \delta_i > 1.$ (i=1, 2,….,n)

Substituting the values of the dual decision variables in the dual objective function of the dual problem we get $v^*(\lambda^*) = \prod_{i=1}^{n} \left(\frac{\alpha_i}{\lambda_{1i}^*} \right)^{\lambda_{1i}^*} \left(\frac{\gamma_i \alpha_i^{1-\delta_i}}{\lambda_{2i}^*} \right)^{\lambda_{2i}^*}$. Following Duffin, Peterson and Zener (1967) we get the optimum value of the primal objective function as $TC_2^* = v^*(\lambda^*)$.

The decision variables of the primal problem can be obtain from the following primal-dual relationships

$$\frac{\alpha_i M_i^{1+\beta_i}}{\lambda_{1i}^*} = v^*(\lambda^*)$$

Solution of the above equation, we get $M_i^* = \left(\frac{\lambda_{1i}^* v^*(\lambda^*)}{\alpha_i} \right)^{\frac{1}{1+\beta_i}}$ (i=1,2,….,n).

The ideal objective value TC_2 is defined as $TC_2 = TC_2^*(M^*)$.

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Source of support: "DST PURSE" in the Department of Mathematics, University of Kalyani, Kalyani, Nadia-741235, India. Conflict of interest: None Declared.

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