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UNSTEADY MHD COUETTE FLOW WITH HEAT TRANSFER OF A COUPLE STRESS FLUID UNDER EXPONENTIAL DECAYING PRESSURE GRADIENT

V. P. Rathod[1] and Syeda Rasheeda Parveen*[2]

[1]Department of Mathematics CNCS, Haramaya University, Ethiopia.

[2]Department of Studies and Research in Mathematics, Gulbarga University, Gulbarga - 585 106, Karnataka, India.

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ABSTRACT

The unsteady magnetohydromagnetic couette flow of an electrically conducting incompressible couple stress fluid between two parallel horizontal non-conducting porous plates is studied with heat transfer. A sudden uniform and an exponential decaying pressure gradient, an external uniform magnetic field that is perpendicular to the plates and uniform suction and injection through the surface of the plates are applied. The two plates are kept at different but constant temperatures while the joule and viscous dissipations are taken into consideration. Numerical solutions for the governing momentum and energy equations are obtained using transform technique (Cosine and Hankel transformation). The effect of magnetic field, couple stress parameter, and velocity of suction and injection on both the velocity and temperature distributions are examined.

Key–words: *MHD flow, Heat transfer, Couple stress, Electrically conducting fluids.*

INTRODUCTION

The flow of an electrically conducting couple stress fluid between two parallel plates in the presence of a transversely applied magnetic field has applications in many devices such as magnetohydrodynamic (MHD) power generators. MHD pumps, accelerators, aerodynamics heating, electrostatic, precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays. Hartmann flow of a Newtonian fluid with heat transfer, subjected to different physical effects, have been studied by many authors [1-9]. These result are important for design of the duct wall and cooling arrangements.

The rectangular channel problem has later been extended also to fluid obeying non-Newtonian constitutive equations. There have been many articles dealing with theoretical modeling and experimental measurements of the particle-phase viscosity in a dusty fluid,Soo[10], and Gidaspow *et.al*, [11]. Couple stress fluid theory proposed by Stokes [12] is the simplest polar fluid theory that shows all important features and effects of couple stress in fluids caused due to the mechanical interactions that occur inside a deforming continuum. Singh and Pathak [13] have discussed unsteady flow of a dusty viscous fluid through a uniform pipe with sector of a circle as cross-section, and pulsatile flow of blood with micro-organism through a uniform pipe with sector of a circle as cross-section in the presence of transverse magnetic field has been investigated by Rathod and Parveen [14]. Pulsatile flow of blood in two layer model in the presence of micro-organism has been investigated by Rathod and Gayatri [15-16]. Also unsteady flow of a dusty magnetic conducting couple stress fluid through a pipe and the flow of a conducting fluid in a circular pipe has been investigated by many authors. Gudiraju *et.al*, [17], Dube and Sharma [18], Ritter and Peddieson [19], Chamkha [20], investigated Steady two phase vertical flow in a pipe. Dube and Sharma [18] and Ritter and Peddieson [19] have reported solutions for unsteady dusty -gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stress. Rathod and Baderunisa [21] have studied by the pulsatile flow of blood in capillaries of small exponential divergence with volume fraction of micro-organism. Rathod *et.al*, [22] have reported solution for coutte flow of conducting dusty visco-elastic fluid through two flat plate under the influence of transverse magnetic field. Rathod and Rasheeda [23, 24] investigated unsteady flow of a dusty magnetic conducting couple stress fluid through a circular pipe and ion slip effect on the unsteady flow of a dusty couple stress fluid through a circular pipe.

Corresponding Author: Syeda Rasheeda Parveen[2], [2]Department of Studies and Research in Mathematics, Gulbarga University, Gulbarga - 585 106, Karnataka, India.*

In most cases the Hall term was ignored in applying Ohm's law as it has no marked effect for small and moderate values of magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable [4] under these condition, the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term.

Attia [8, 9] has studied the influence of the magnetic field on the velocity and temperature field of an unsteady Hartmann flow of a conducting Newtonian fluid between two infinite non-conducting horizontal parallel fixed and porous plates.

In the present work MHD **c**ouette flow of a conducting couple stress fluid is studied with heat transfer in the presence of a uniform magnetic field and exponential decaying pressure gradient. The upper plate is moving with constant velocity while the lower plate is kept stationary. The flow is subjected to a uniform suction from above and a uniform injection from below and an external uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4]. The two plates are kept at two different but constant temperatures. The Joule and viscous dissipations are taken into consideration in the energy equation.The governing momentum and energy equation is solved by using transform technique (Cosine and Hankel transform [25]). The inclusion of magnetic field, The non-Newtonian fluid, characteristic as well as the velocity of suction and injection leads to some interesting effects and also couple stress parameter, on both velocity and temperature field.

FORMULATION OF THE PROBLEM

The fluid is assumed to be incompressible, couple sterss and flows between two infinite horizontal parallel non-conducting plates located at the d = $\pm$ h planes and extend from x = - ∞ to ∞ and from z = -∞ to ∞**.** The upper plate is moving with a constant velocity u_o while the lower plate is kept stationary. The upper and lower plates are kept at two constant temperature T_2&T_1respectively,with T_2>T_1.The flow is driven by a uniform and exponentially decaying pressure gradient in the x-direction and a uniform suction from the above and injection from below which are applied at t = o. A uniform magnetic field with magnetic flux density rector B_o is applied in positive y-direction which is assumed to be the total magnetic field ,as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4]. The plates are assumed to be infinite in the x and z-directions which makes the physical quantities do not change in these directions. Thus the velocity vector of the fluid, in general ,is given by

$$v(y, t) = u(y, t)i + v_o j$$

where u(y, t) is the main velocity component in the x-direction. It is because of the conservation of mass, i.e., div v = o and due to the uniform suction the velocity component v(y, t) is assumed to have a constant value v_o in the y – direction.

The fluid motion starts from rest at t=o that is u=o for t $\leq$ o, the no-slip condition at the plates implies that u=o, at y=-h and u=u_o, at y=h. It is also assumed that the initial temperature of the fluid is T_1, thus the initial and boundary conditions of temperature at T=T_1 at t=o, T=T_2 at y=-h, t>o, and T=T_2, y=h, t>o. The fluid motion is governed by the momentum equation [26]

$$\rho\left(\frac{\partial u}{\partial t} + (\gamma_0)\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \eta\frac{\partial^4 u}{\partial y^4} + \sigma B_0^2 u + \mu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

where ρ: density of the fluid .

σ: electric conduction of the fluid.

$\sigma B_0^2 u$: electromagnetic force.

η: couple Stress parameter.

The temperature distribution is governed by the energy equation [26]

$$\rho c_p\left(\frac{\partial T}{\partial t} + \gamma_0\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \sigma B_0^2 u^2 + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2}$$

C_p: specific heat capacity, k: thermal conductivity of the fluid.

The second and third term on the right- hand side represent the Joule and viscous dissipations respectively. Equations (1) and (2) can be made dimensionless by introducing the following dimensionless variables and parameters,

$\hat{x} = \frac{x}{h}$, $\hat{y} = \frac{y}{h}$, $\hat{u} = \frac{u}{U_o}$, $\hat{p} = \frac{P}{\varrho u_o^2}$, $\hat{t} = \frac{tU_o}{h}$

$\hat{T} = \frac{T-T_1}{T_2-T_1}$, $R_e = \frac{\varrho h U_o}{\mu}$: is the Reynolds number

s = $\frac{\gamma_0}{U_o}$: is the suction parameter, $P_r = \frac{\mu c_p}{k}$: is the Prandtl number,

$E_c = \frac{U_o^2}{C_p(T_2-T_1)}$: is the Eckert number, $H^2{}_a = \frac{\sigma B_o^2 h^2}{\mu}$: is the Hartmann number,

$\bar{\alpha}^2 = \frac{h^2 m}{\eta}$: is the Couple stress parameter

Equation (1) and (2) after droping caps for convenience

$$\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} = -\frac{dp}{d_2} - \frac{1}{Re} H^2{}_a u - \frac{1}{\alpha^2}\frac{1}{Re}\frac{\partial^4 u}{\partial y^4} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} \tag{3}$$

Equation (2) becomes.

$$\frac{\partial T}{\partial t} + s\frac{\partial T}{\partial y} = \frac{1}{Re\, P_r}\frac{\partial^2 T}{\partial y^2} + \frac{H^2{}_a}{Re}\, E_c\, u^2 + \frac{Ec}{Re}\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

The initial and boundary condition for the velocity and temperature in the dimensionless from are written as.

$$u(y, o) = o, \quad u(-1, t) = o, \quad u(1, t) = 1 \tag{5}$$

$$T(y, o) = o, \quad T(-1, t) = o, \quad T(1, t) = 1 \tag{6}$$

where the exponential decaying pressure gradient is assumed in the form

$$\frac{-dp}{dx} = c\, e^{-\beta t} \ .$$

Applying cosine transform to (3) and (4)

$$\frac{\partial^2 \bar{u}}{\partial t^2} + x_1\frac{\partial \bar{u}}{\partial t^2} + x_2\bar{u} = x_3. \quad x_1 = -S - \frac{1}{\bar{\alpha}^2 R_e} - -\frac{1}{R_e}, \tag{7}$$

$$x_2 = -\frac{SHa^2}{Re} - \frac{(Ha^2)^2}{Re^2} + \frac{Ha^2}{\propto^{-2} R_e^2} - \frac{Ha^2}{Re}, \quad x_3 = (\beta c - \frac{c}{\beta}Ha^2)e^{-\beta t}\frac{(-1^m)}{2m+1}(2\alpha)$$

$$\frac{\partial \bar{T}}{\partial T} + s(\bar{T}) = \frac{1}{RePr}\bar{T} + \frac{Ha^2 Ec}{Re}u^2 + \frac{Ec}{Re}(\bar{u})^2 \tag{8}$$

Applying Hankel transform to (7) and (8)

$\bar{\bar{u}} = \int_0^1 \bar{u}\,(r, y, t)\, r\, J_o\,(r\,\mathcal{E}i)dr$; $J_o(\mathcal{E}i) = o$, $\mathcal{E}i$: are the +ve roots of Bessels function,

$$\propto_1 = \frac{1}{2}\left[-x_1 + \sqrt{x_1^2 - 4x_2}\right], \quad \propto_2 = \frac{1}{2}\left[-x_1 - \sqrt{x_1^2 - 4x_2}\right],$$

$$\bar{\bar{u}} = \frac{\left(-\beta c - \frac{1}{Re}Ha^2\right)\sum_{m=0}^{\infty}\frac{(-1)^m}{n}}{\beta^2 - \beta x_2 + x_2}\left\{e^{-\beta t} + \frac{[(\propto_2+\beta)e^{\propto_1 t} - (\propto_1+\beta)e^{\propto_2 t}]}{\sqrt{x_1^2 - 4x_2}}\right\}\sum_{i=1}^{\infty}\frac{J_1\,(r\mathcal{E}i)}{\varepsilon_i} \tag{9}$$

$$\frac{\partial \bar{\bar{T}}}{\partial T} - (-s + \frac{1}{RePr})\,\bar{\bar{T}} = \frac{Ec}{Re}\,(H_a{}^2 + 1)\,\bar{\bar{u}}^2 \tag{10}$$

Applying inverse Hankel transform and inverse cosine transform to (9) and (10)

$$u = \frac{4}{\propto}\sum_{m=o}^{\infty}\frac{(-1)^m}{\beta^2 - \beta x_1 + x_2}\left[-\beta c - \frac{1}{Re}Ha^2\right]\sum_{i=1}^{\infty}\frac{1}{\mathcal{E}i}\frac{J_o\,(r\mathcal{E}i)}{J_1(\mathcal{E}i)} - \cos n\bar{\wedge}y.\left\{e^{-\beta t} + \frac{[(\propto_2+\beta)e^{\propto_1 t} - (\propto_1+\beta)e^{\propto_2 t}]}{\sqrt{x_1^2 - 4x_2}}\right\} \tag{11}$$

$$T = Y_2\left\{\begin{array}{l} \frac{1}{-2\beta - y_1}\left(e^{-2\beta t} - e^{y_1 t}\right) + \frac{1}{(x_1^2 - 4x_2)}\frac{[(\propto_2+\beta)(e^{2\propto_1 t} - e^{y_1 t})]}{2\propto_1 - y_1} + \frac{[(\propto_1+\beta)(e^{2\propto_2 t} - e^{y_1 t}]}{2\propto_2 - y_1} \\ -\frac{2(\propto_2+\beta)(\propto_1+\beta)}{(\propto_2+\propto_1) - y_1}\left(e^{(\propto_1+\propto_2)t} - e^{y_1 t}\right)] + \frac{2}{\sqrt{(x_1^2 - 4x_2)}}\left\{\frac{(\propto_2+\beta)(e^{(\propto_1-\beta)t} - e^{y_1 t})}{\propto_1 - \beta - y_1} - \frac{(\propto_1+\beta)(e^{(\propto_2+\beta)t} - e^{y_1 t})}{\propto_2 - \beta - y_1}\right\} \end{array}\right\} \tag{12}$$

where $Y_2 = \frac{Ec(Ha^2+1)}{Re}\left\{\frac{16}{\propto^2}\sum_{m=o}^{\infty}\frac{(-1)^{2m}\,[-\beta c + \frac{1}{Re}Ha]^2}{(\beta^2 - \beta x_1 + x_2)^2 n^2}\right\}\left[\sum_{i=1}^{\infty}\frac{1}{\mathcal{E}i}\frac{J_o\,(r\mathcal{E}i)}{J_1(\mathcal{E}i)}\cos n\,\bar{\wedge}\,y\right]^2$,

$Y_1 = -s + \frac{1}{RePr}$,

Computations have been made for c = 5, β = 1, P_r = 1, Re = 1, Ec = 0.2, plotted the graph for different values of couple stress parameter, Hartmann number, suction parameter and time by using "Mathematica".

RESULT AND DISCUSSION

The profiles of the velocity u and temperature T respectively for various values of time t and for couple stress parameter $\bar{\alpha}^2$ =q=O.5, 1, 1.5; for H_a=1,S=1 have been computed ,it is observed that increasing couple stress parameter decrease velocity and temperature at small time remains steady and then decrease at large time in fig 1-2

Figure 3-4, shows that the time evaluation of velocity and temperature at the centre of the channel (y=0.0), respectively for various values of the parameter H_a^2=0, 1, 3 for couple stress parameter q= 0.5, 1, 1.5. In these figure 3, when suction parameter is zero, with Hartmann number decrease velocity for all values of couple stress parameter q. This is because the magnetic damping force on u increase with increasingH_a^2. Figure 4 indicates that the effect of the parameter Hartmann number on temperature depends on time, increasing H_a^2 increases T at small times but decreases at large times. This can be attributed to the fact that for small times on increase in H_a^2 increases the Joule dissipations with is proportional to H_a^2. For large times, increasing H_a decreases u greatly and in turn decreases the Joule and viscous dissipations. This accounts for crossing the curves of T with the time for all values of couple stress .Figures 4b and 4c shows that such a crossover becomes more apparent for higher couple stress parameter. The parameter q has a negligible effect on the steady state time of T for all values of H_a^2.

Figure 5-6 shows the time profiles of velocity and temperature centre of the channel y=0.0 respectively for various values of the suction parameter S= (0, 1, 3) and for q= 0.5, 1, 1.5. In these figures H_a^2=0. Figures-5 shows that increasing the suction decrease velocity due to the convection of fluid from region in the lower half to the centre which higher fluid speed. The parameter q has a marked effect on the steady state time of velocity for all Values of S. If q increase by all values of S then u is increase.

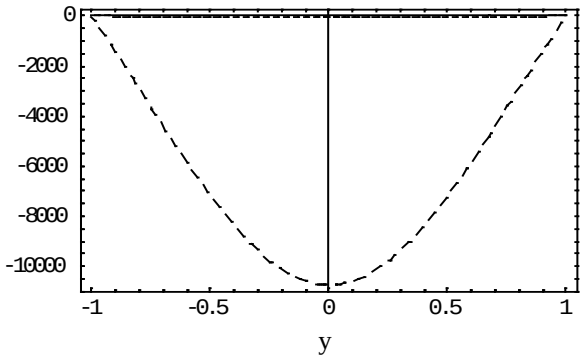
Figure 6 shows that increasing S decreases the temperature at the centre of channel for the all values of q and t. This is due to the influence of the convection in pumping the fluid from the cold lower half towards the centre of the channel. The effect of S and T has apparent effects for all q.

Figure 7-8 shows the time evolution of u and T at the centre of the various channel respectively for various values of the couple stress parameter q = 0.5, 1, 2. The figures are evaluated for H_a^2== 1, S=1, figures 7 & 8 indicates that increasing q increases u and T but decrease at large time. Also it is shown that increase couple stress parameter increase velocity and temperature due to the exponential decaying pressure gradient.

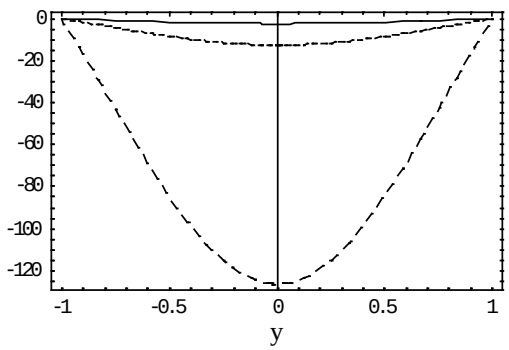
CONCLUSION

The unsteady MHD couple stress flow and heat transfer of conducting under the influence of an applied uniform magnetic field have been studied in the presence of a uniform magnetic field as well as a uniform suction and injection. The effect of magnetic field, couple stress parameter, suction and injection on the velocity and temperature distributions has been investigated. The couple stress parameter has more effect on the velocity and temperature distributions and their steady state times for all values of the magnetic field and suction velocity. The variation of the velocity and temperature with the couple stress parameter was shown to depend on time. Also ,it is of interest to know that the dependence of the temperature on the magnetic field vary with time for all values of couple stress parameter

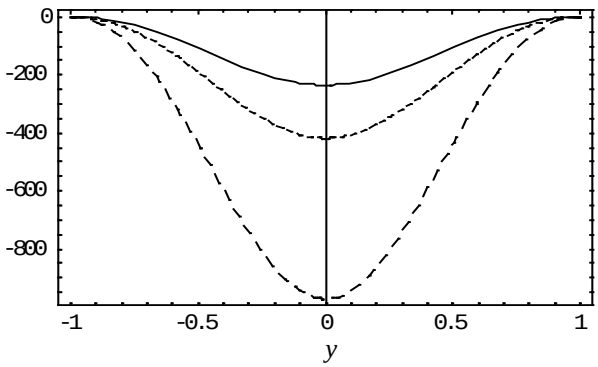
Figure 1-2. Effect of the parameter t on u Tfor various values of q. S=0, H_a^2==0



(a)_____for t=0.5,……for t=1,----for t=2
q=0.5

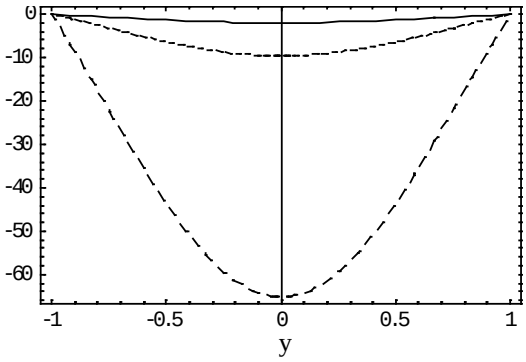


(a)_____for t=0.5,……for t=1,----for t =2
q=0.5



(b)_____for t=0.5,……for t=1,----for t=2

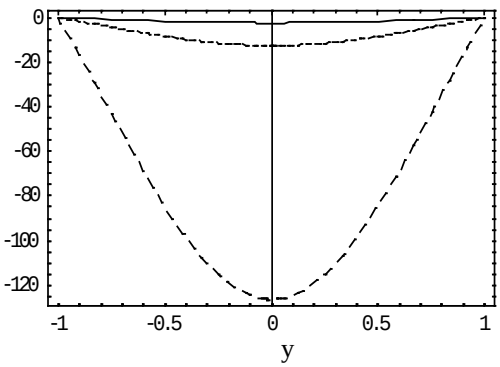
q=1



(c)_____for t=0.5,……for t=1----for t=2

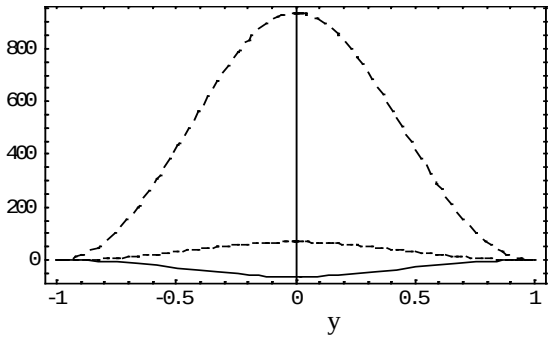
q=1.5

Fig. 1



(b)_____for t=0.5,……for t=1,----for t =2

q=1

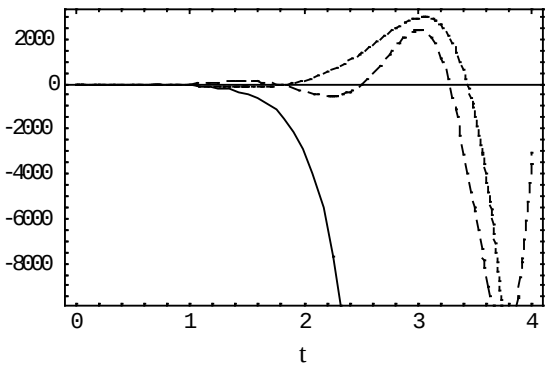


(c)_____for t=0.5,……for t=1,----for t =2

q=1.5

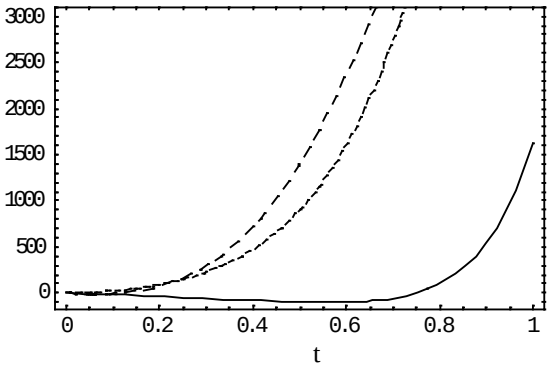
Fig. 2

Figure 3-4. Effect of the parameter H_a on u and Tfor various values of q. S=0



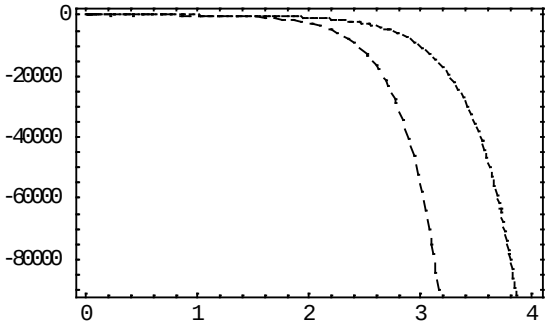
(a)____for H_a=1,……for H_a=2,---for H_a=3

q=0.5



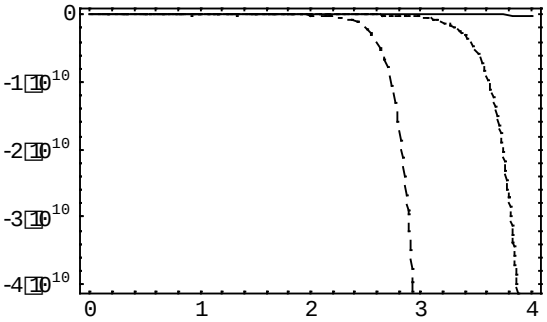
(a)____for H_a=1,…for H_a=2,----for H_a=3

q=0.5



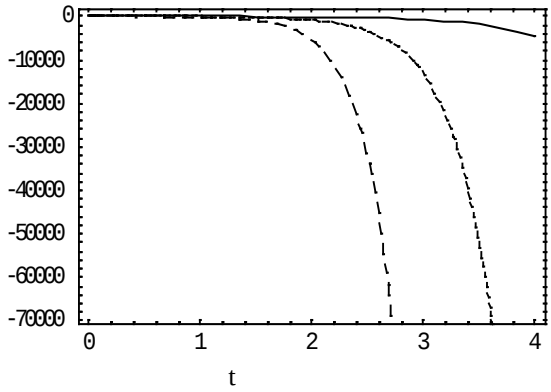
(b)_____for H_a=1,……for H_a=2,----for H_a=3

q=1



(b)_____for H_a=1,……forH_a=2,--forH_a=3

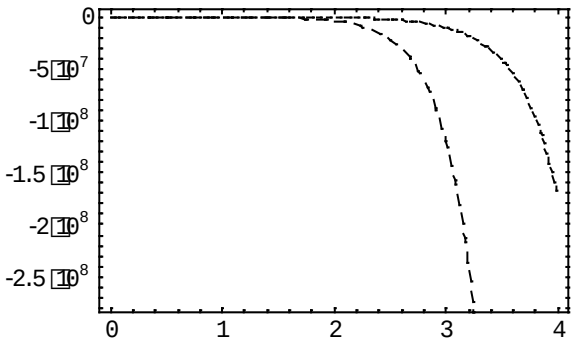
q=1



(c)_____for H_a=1,……forH_a=2,----for H_a=3

q=1.5

Fig. 3

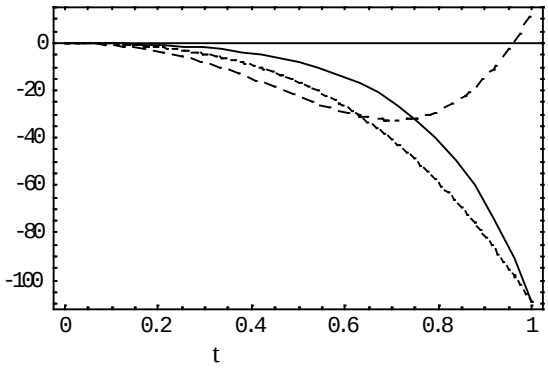


(c)_____for H_a=1,…… forH_a=2,----for H_a=3

q=1.5

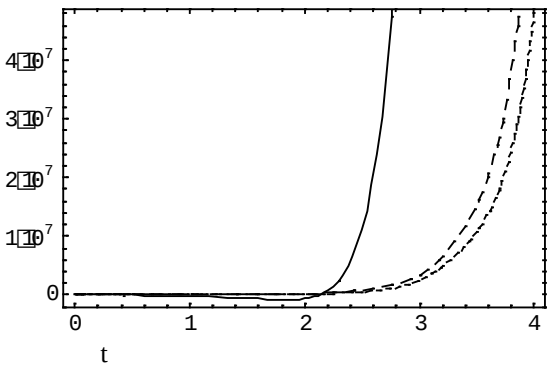
Fig. 4

Figure 5-6. Effect of the parameter S on u and T for various values of q. H_a=o



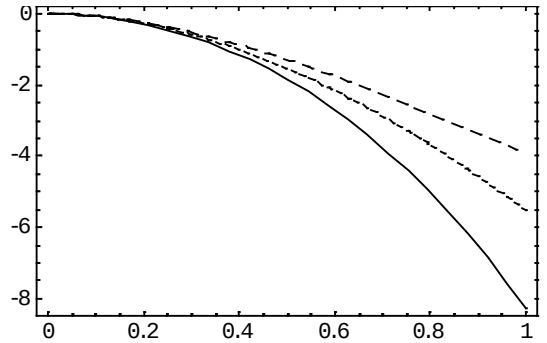
(a)_____for S=1,……for S=2,----for S=3

q=0.5 , (a)



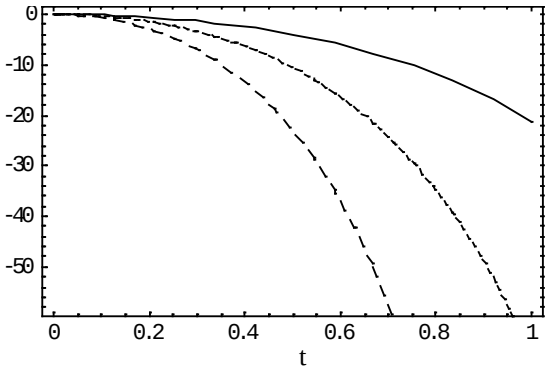
(a)_____for S=1……for S=2,--for S=3

q=0.6



(b)_____for S=1,……for S=2,----for S=3

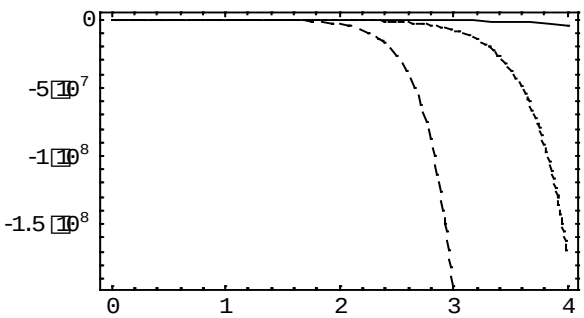
q=1



(c)_____for S=1,……for S=2,----for S=3

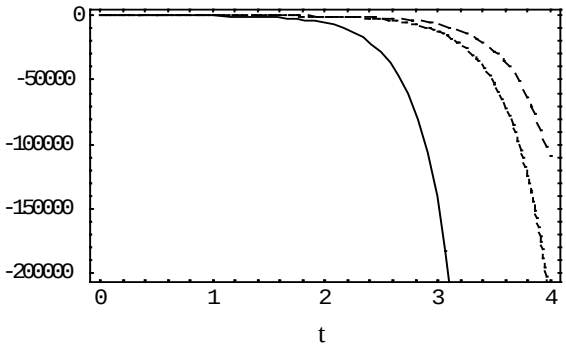
q=1.5

Fig. 5



(b)____for S=1,…for S=2,----for S=3

q=1

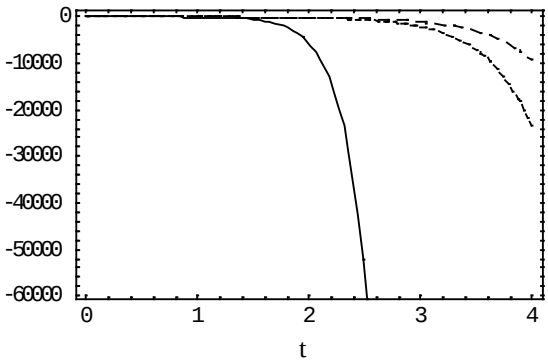


(c)_____for S=1,……for S=2,----for S=3

q=1.5

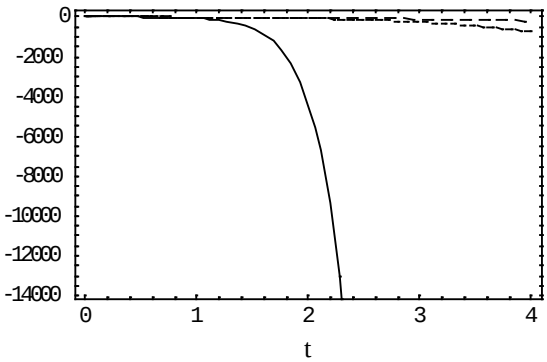
Fig. 6

Figure 7. Effect of the parameter q on u for S=0, H_a=o and S=1, H_a =1



(a)_____for q=0.5,…for q=1,----for t=2

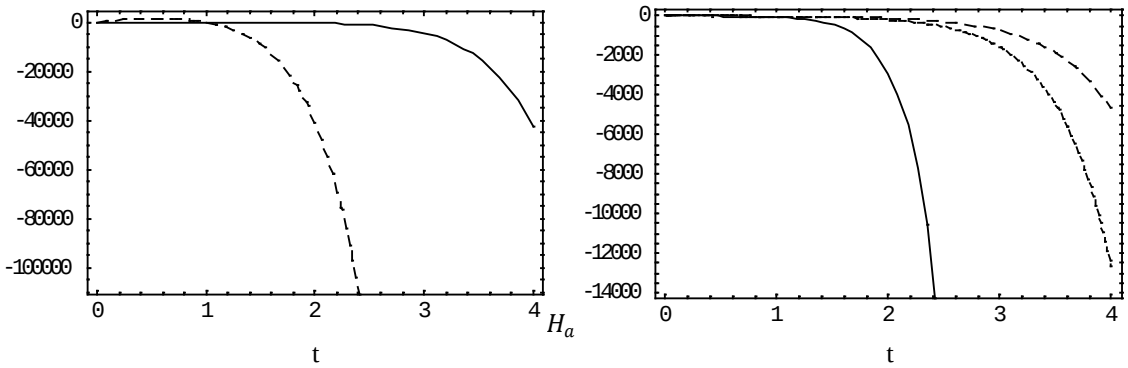
S=0,h=0



(b)_____for q=0.5,……for q=1,---forq=2

S=1,h=1

Figure 8. Effect of the parameter q on T for s=0. H_a ,h=o and s=1, H_a=1



(a)_____for q=0.5,…for q=1,--for q=2
S=0,H_a=0

(b)_____for q=0.5,…for q=1,----for q=2
S=1,H_a=1

REFERENCES

1. Tao.1.N, J of Aerospace Sci, Vol. 27, (1980).
2. Alpher R.A, Int –J Heat and Mass transfer Vol. 3, (1961).
3. Tani 1. J of Aerospace Sci. Vol. (29). p.287 (1962).
4. Sutton G.W. and Sherman. A Engineering MHD Me-Graw-Hill New York (1965).
5. Soundalgekar V.M. IEEE transactions on plasma science Vol. Ps.7. P-178 (1979).
6. Soundalgekar V.M. and Uplekar A.G. IEEE transaction on plasma Science.Vol. PS-14, 579, (1986).
7. Attia H.A. and Kotb N.A. Acta Mechanic Vol. 117, P-215, (1996).
8. Attia H.A. Can J. Phys Vol. 76, P-739, (1998).
9. Attia H.A. Mechanic Res. Comm. Vol. 26, P-115. (1999).
10. Soo,S L , Sci Res,vol 21,pp 68-84.(1969).
11. Gidaspow,D., Shih,Y.T.,Bouillard, J. and Wasan, D., , AIChE Journal, v.35, n.5, p.715-724. (1989)
12. Stokes,v.k. The physics of fluid 3, (9) p 1709-1715(1966).
13. Singh and Pathak, .Ind.J,of pure and Appl. Math 8(6) p 695-701(1977).
14. Rathod V.P. and Parveen S.R. Math Edn. 3, p 121-133. (1997).
15. Rathod V.P. and Gayathri PART-II''.
16. Rathod V.P. and Gayatri, Maths Edn. 35(3)p 150-160. (2001).
17. Gudiraju, M., Peddieson, J. and Munukutle,s..Mechanic Research Communication, 19,p 7-13(1992).
18. Dube,S.N.and Sharma,C.L., J.phy. soc. Japan, 38,p 298-310,(1975).
19. Ritter, J.M and peddieson,J.. Proceeding of the 6th Canadian congress of Applied Mechanics, (1977).
20. Chamkha, A J. Mechanic Research Communication, vol.21 (3).pp 281-286 (1994).
21. Rathod V.P. and Baderunnisa Begum. Int. J. Mathematical Science and Engineering Application 4(4) pp 229-241. (2010).
22. Patel G.S. Rathod V.P. and Haq K.A. Sci & Tech. Res. J. GUG.3, (1990).
23. Rathod V.P. and Syeda Rasheeda Parveen. Int. Mathematical Science and Engineering Application vl.8, No.II, 149-160. (2014).
24. Rathod V.P. and Syeda Rasheeda Parveen. Int. J. Mathematical Science and Engineering Application Vol.8, No. IV, 189-194. July (2014).
25. Sneddon I.N. "Fourier Transforms" Mc. Grow-Hill Book, Co. New York 1951.
26. Schlichting, H., Boundary layer theory, McGraw-Hill, New York (1986).

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