

**EFFECT OF MAGNETIC FIELD ON UNSTEADY CONVECTIVE HEAT AND MASS TRANSFER
FLOW IN A VERTICAL WAVY CHANNEL WITH OSCILLATORY FLUX
WITH RADIATION ABSORPTION**

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ABSTRACT

We discuss the effect of chemical reaction on unsteady free convective heat and mass transfer flow in a vertical wavy channel under the influence of a transverse magnetic field. The unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope δ as the perturbation parameter. The expression for the velocity, the temperature, the concentration, the shear stress and the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters M , β , k and Q_1 .

Keywords: *Magnetic Field, Heat and Mass transfer, Vertical Wavy Channel, Radiation, Oscillatory Flux.*

1. INTRODUCTION

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage, evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[2], Lai and Kulacki[13] and Murthy and Singh[18]. Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai [12]. The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa *et al.* [1]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood [21,22], Lee *et al.* [15] and others [33,36].

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established [8] that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging – diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh [34] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry [35] have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath [36] have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMichael and Deutsch [16], Deshikachar *et al* [6], Rao *et al.* [25] and Sree Ramachandra Murthy [32]. Hyan Goo Kwon *et al.* [11] have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Comini *et al.* [4] have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers.

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In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Das *et al.* [7] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumara-swamy [20] has studied the effects of reaction on a long surface with suction. Recently Gnaneswar [9] has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

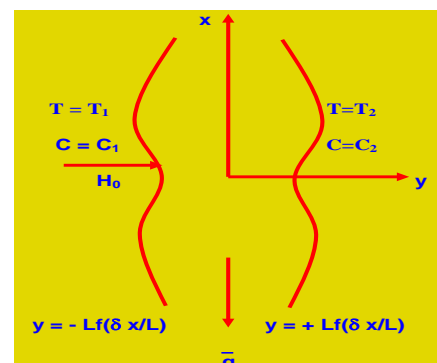
The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simultaneous heat and mass transfer from different geometrics embedded in a porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in much industrial application such as geophysics, oceanography, drying process, solidification of binary alloy and chemical engineering. Kandaswamy *et al.* [14] have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.

Recently Madhusudan Reddy [17], has analysed the effect of chemical reaction on double diffusive heat transfer flow of a viscous fluid in a wavy channel.

In this paper, we discuss the effect of chemical reaction on unsteady free connective heat and mass transfer flow in a vertical wavy channel under the influence of a transverse magnetic field. The unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope δ as the perturbation parameter. The expression for the velocity, the temperature, the concentration, the shear stress and the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters M , β , k and Q_1 .

2. FORMULATION OF THE PROBLEM

We consider the effect of chemical reaction on the unsteady motion of viscous, incompressible fluid in a vertical channel bounded by wavy walls. The thermal buoyancy in the flow field is created by an oscillatory flux in the fluid region. The walls are maintained at constant temperature and concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are neglected in comparison with heat by conduction and convection in the energy equation. Also the Kinematic viscosity ν , the thermal conducting k are treated as constants. We choose a rectangular Cartesian system $O(x, y)$ with x -axis in the vertical direction and y -axis normal to the walls. The walls of the channel are at $y = \pm Lf\left(\frac{\delta x}{L}\right)$.



The equations governing the unsteady flow, heat and mass transfer are Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Equation of linear momentum

$$\rho_e \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g - (\sigma \mu_e^2 H_o^2) u \quad (2)$$

$$\rho_e \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

Equation of Energy:

$$\rho_e C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - Q(T - T_e) + Q_1(C - C_e) \quad (4)$$

Equation of diffusion

$$\left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_1 \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - k_1(C - C_e) \quad (5)$$

Equation of state

$$\rho - \rho_e = -\beta \rho_e (T - T_e) - \beta^* \rho_e (C - C_e) \quad (6)$$

where ρ_e is the density of the fluid in the equilibrium state, T_e , C_e are the temperature and concentration in the equilibrium state, (u, v) are the velocity components along $O(x, y)$ directions, p is the pressure, T , C are the temperature and Concentration in the flow region, ρ is the density of the fluid, μ is the constant coefficient of viscosity, C_p is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, k is the permeability of the porous medium, β is the coefficient of thermal expansion, Q is the strength of the constant internal heat source, σ is the electrical conductivity, μ_e is the magnetic permeability, β^* is the volumetric expansion with mass fraction coefficient D_1 , is the molecular diffusivity and k_1 is the chemical reaction coefficient.

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \quad (7)$$

where $p = p_e + p_D$, p_D being the hydrodynamic pressure.

The flow is maintained by an oscillatory volume flux for which a characteristic velocity is defined as

$$q(1 + k e^{i\omega t}) = \frac{1}{L} \int_{-L_f}^{L_f} u dy. \quad (8)$$

The boundary conditions for the velocity and temperature fields are

$$\begin{aligned} u = 0, v = 0, T = T_1, C = C_1 \quad \text{on } y = -Lf \left(\frac{\delta x}{L} \right) \\ u = 0, v = 0, T = T_2, C = C_2 \quad \text{on } y = +Lf \left(\frac{\delta x}{L} \right) \end{aligned} \quad (9)$$

In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y, v = \psi_x \quad (10)$$

Eliminating pressure p from equations (2) & (3) and using the equations governing the flow in terms of ψ are

$$[(\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x] = \nu \nabla^4 \psi - \beta g (T - T_0)_y - \beta^* g (C - C_0)_y - (\sigma \mu_e^2 H_o^2) \frac{\partial^2 \psi}{\partial y^2} \quad (11)$$

$$\rho_e C_p \left(\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \lambda \nabla^2 T - Q(T - T_0) + Q_1(C - C_e) \quad (12)$$

$$\left(\frac{\partial C}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = D \nabla^2 C - k_1 (C - C_o) \quad (13)$$

Introducing the non-dimensional variables in (9) & (10) as

$$x' = x/L, \quad y' = y/L, \quad t' = t\omega, \quad \Psi' = \Psi/\nu, \quad \theta = \frac{T - T_2}{T_1 - T_2}, \quad C' = \frac{C - C_2}{C_1 - C_2} \quad (14)$$

The governing equations in the non-dimensional form (after dropping the dashes) are

$$R \left(\gamma^2 (\nabla^2 \psi)_t + \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} \right) = \nabla^4 \psi + \left(\frac{G}{R} \right) (\theta_y + N C_y) - M^2 \frac{\partial^2 \psi}{\partial y^2} \quad (15)$$

$$P \left(\gamma^2 \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta - \alpha \theta + Q_1 C \quad (16)$$

$$Sc \left(\gamma^2 \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla^2 C - KC \quad (17)$$

where

$$\begin{aligned} R &= \frac{UL}{\nu} & (\text{Reynolds number}), & \quad G = \frac{\beta g \Delta T_e L^3}{\nu^2} & (\text{Grashof number}) \\ P &= \frac{\mu c_p}{k_1} & (\text{Prandtl number}), & \quad M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{\nu^2} & (\text{Hartmann Number}) \\ Sc &= \frac{\nu}{D_1} & (\text{Schmidt Number}), & \quad \alpha = \frac{QL^2}{\lambda} & (\text{Heat source parameter}) \\ Q_1 &= \frac{Q_1' L^2 \Delta C_e}{\lambda \Delta T_e} & (\text{Radiation absorption parameter}), & \quad K = \frac{K_1 L^2}{D_1} & (\text{Chemical reaction parameter}) \\ \gamma^2 &= \frac{\omega L^2}{\nu} & (\text{Womersley Number}), & \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \end{aligned}$$

The corresponding boundary conditions are

$$\begin{aligned} \psi(+f) - \psi(-f) &= 1 \\ \frac{\partial \psi}{\partial x} &= 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm f \end{aligned} \quad (18)$$

$$\begin{aligned} \theta(x, y) &= 1, C = 1 \quad \text{on } y = -f \\ \theta(x, y) &= 0, C = 0 \quad \text{on } y = f \\ \frac{\partial \theta}{\partial y} &= 0, \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0 \end{aligned} \quad (19)$$

The value of ψ on the boundary assumes the constant volumetric flow in consistant with the hyphothesis (7). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t .

3. METHOD OF SOLUTION

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

Introduce the transformation such that

$$\bar{x} = \delta x, \frac{\partial}{\partial x} = \delta \frac{\partial}{\partial \bar{x}}$$

Then $\frac{\partial}{\partial x} \approx O(\delta) \rightarrow \frac{\partial}{\partial \bar{x}} \approx O(1)$

For small values of $\delta \ll 1$, the flow develops slowly with axial gradient of order δ And hence we take $\frac{\partial}{\partial \bar{x}} \approx O(1)$

Using the above transformation the equations (15-17) reduces to

$$\delta R \left(\gamma^2 (\nabla_1^2 \psi)_t + \frac{\partial(\psi, \nabla_1^2 \psi)}{\partial(x, y)} \right) = \nabla_1^4 \psi + \left(\frac{G}{R} \right) (\theta_y + NC_y) - M^2 \frac{\partial^2 \psi}{\partial y^2} \quad (20)$$

$$\delta P \left(\gamma^2 \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla_1^2 \theta - \alpha \theta + Q_1 C \quad (21)$$

$$\delta Sc \left(\gamma^2 \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla_1^2 C - KC \quad (22)$$

where

$$\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Introducing the transformation

$$\eta = \frac{y}{f(\bar{x})}$$

The equations (20-22) reduces to

$$\delta R f \left(\gamma^2 (F^2 \psi)_t + \frac{\partial(\psi, F^2 \psi)}{\partial(\bar{x}, \eta)} \right) = F^4 \psi + \left(\frac{G f^3}{R} \right) (\theta_\eta + NC_\eta) - (M^2 f^2) \frac{\partial^2 \psi}{\partial \eta^2} \quad (23)$$

$$\delta P \left(\gamma^2 \frac{\partial \theta}{\partial t} + f \left(\frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \eta} \right) \right) = F^2 \theta - (\alpha f^2) \theta + (Q_1 f^2) C \quad (24)$$

$$\delta Sc \left(\gamma^2 \frac{\partial C}{\partial t} + f \left(\frac{\partial \psi}{\partial \eta} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial \eta} \right) \right) = F^2 C - K f^2 C \quad (25)$$

where $F^2 = \delta^2 \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \eta^2}$

We adopt the perturbation scheme and write

$$\begin{aligned} \psi(x, \eta, t) &= \psi_0(x, \eta, t) + ke^{it} \bar{\psi}_0(x, \eta, t) + \delta(\psi_1(x, \eta, t) + ke^{it} \bar{\psi}_1(x, \eta, t)) + \dots \\ \theta(x, \eta, t) &= \theta_0(x, \eta, t) + ke^{it} \bar{\theta}_0(x, \eta, t) + \delta(\theta_1(x, \eta, t) + ke^{it} \bar{\theta}_1(x, \eta, t)) + \dots \\ C(x, \eta, t) &= C_0(x, \eta, t) + ke^{it} \bar{C}_0(x, \eta, t) + \delta(C_1(x, \eta, t) + ke^{it} \bar{C}_1(x, \eta, t)) + \dots \end{aligned} \quad (26)$$

On substituting (20) in (23) - (25) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{0,\eta\eta} = - \left(\frac{G f^3}{R} \right) (\theta_{0,\eta} + NC_{0,\eta}) \quad (27)$$

$$\theta_{0,\eta\eta} - (\alpha f^2) \theta_0 = -(Q_1 f^2) C_0 \quad (28)$$

$$C_{0,\eta\eta} - (K S c f^2) C_0 = 0 \quad (29)$$

with

$$\begin{aligned} \Psi_{0(+1)} - \Psi_{0(-1)} &= 1, \\ \Psi_{0,\eta} &= 0, \quad \Psi_{0,x} = 0 \quad \text{at } \eta = \pm 1 \end{aligned} \quad (30)$$

$$\begin{aligned} \theta_o &= 1, \quad C_o = 1 \quad \text{on } \eta = -1 \\ \theta_o &= 0, \quad C_o = 0 \quad \text{on } \eta = 1 \end{aligned} \quad (31)$$

$$\bar{\theta}_{0,\mu\eta} - (iP\gamma^2 f^2) \bar{\theta}_0 = -(Q_1 f^2) \bar{C}_o \quad (32)$$

$$\bar{C}_{0,\eta\eta} - (KSc\gamma^2 f^2) \bar{C}_o = 0 \quad (33)$$

$$\bar{\psi}_{0,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2) f^2) \bar{\psi}_{0,\eta\eta} = -\left(\frac{Gf^3}{R}\right) (\bar{\theta}_{0,\eta} + N\bar{C}_{0,\eta}) \quad (34)$$

$$\begin{aligned} \bar{\theta}_o(\pm 1) &= 0 \quad \bar{C}_o(\pm 1) = 0 \\ \bar{\psi}_o(+1) - \bar{\psi}_o(-1) &= 1 \quad \bar{\psi}_{o,\eta}(\pm 1) = 0, \bar{\psi}_{o,x}(\pm 1) = 0 \end{aligned} \quad (35)$$

The first order equations are

$$\psi_{1,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{1,\eta\eta} = -\left(\frac{Gf^3}{R}\right) (\theta_{1,\eta} + N C_{1,\eta}) + (Rf) (\psi_{0,\eta} \psi_{0,x\eta\eta} - \psi_{0,x} \psi_{0,\eta\eta\eta}) \quad (36)$$

$$\theta_{1,\eta\eta} - (\alpha f^2) \theta_1 = (PRf) (\psi_{0,x} \theta_{o,\eta} - \psi_{0,\eta} \theta_{ox}) - (Q_1 f^2) C_1 \quad (37)$$

$$C_{1,\eta\eta} - (KScf^2) C_1 = (Scf) (\psi_{0,x} C_{o,\eta} - \psi_{0,\eta} C_{ox}) \quad (38)$$

$$\begin{aligned} \bar{\psi}_{1,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2) f^2) \bar{\psi}_{1,\eta\eta} &= -\left(\frac{Gf^3}{R}\right) (\bar{\theta}_{1,\eta} + N \bar{C}_{1,\eta}) + (Rf) (\bar{\psi}_{0,\eta} \psi_{0,x\eta\eta} \\ &\quad + \psi_{0,\eta} \bar{\psi}_{0,x\eta\eta} - \psi_{0,x} \bar{\psi}_{0,\eta\eta\eta} - \bar{\psi}_{0,x} \bar{\psi}_{0,\eta\eta\eta}) \end{aligned} \quad (39)$$

$$\bar{\theta}_{1,\eta\eta} - ((iP\gamma^2 + \alpha) f^2) \bar{\theta}_1 = (PRf) (\psi_{0,\eta} \bar{\theta}_{o,x} + \bar{\psi}_{0,\eta} \theta_{o,x} - \bar{\psi}_{0,x} \theta_{o,\eta} - \psi_{0,x} \bar{\theta}_{o\eta}) - (Q_1 f^2) \bar{C}_{1,\eta\eta} \quad (40)$$

$$\bar{C}_{1,\eta\eta} - ((K + i\gamma^2) Scf^2) \bar{C}_1 = (Scf) (\psi_{0,\eta} \bar{C}_{o,x} + \bar{\psi}_{0,\eta} C_{o,x} - \bar{\psi}_{0,x} C_{o,\eta} - \psi_{0,x} \bar{C}_{o\eta}) \quad (41)$$

with

$$\begin{aligned} \Psi_{1(+1)} - \Psi_{1(-1)} &= 0 \\ \Psi_{1,\eta} &= 0, \quad \Psi_{1,x} = 0 \quad \text{at } \eta = \pm 1 \\ \theta_1(\pm 1) &= 0 \quad C_1(\pm 1) = 0 \end{aligned} \quad (42)$$

$$\begin{aligned} \bar{\theta}_1(\pm 1) &= 0 \quad \bar{C}_1(\pm 1) = 0 \\ \bar{\psi}_1(+1) - \bar{\psi}_1(-1) &= 1 \quad \bar{\psi}_{1,\eta}(\pm 1) = 0, \bar{\psi}_{1,x}(\pm 1) = 0 \end{aligned} \quad (43)$$

4. NUSSELT NUMBER and SHERWOOD NUMBER

$$(\tau)_{y=-1} = d_6 + Ecd_7 + \delta d_8 + O(\delta^2)$$

The local rate of heat transfer coefficient(Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y} \right)_{\eta=\pm 1} \quad \text{where} \quad \theta_m = 0.5 \int_{-1}^1 \theta d\eta$$

and the corresponding expressions are

$$(Nu)_{\eta=+1} = \frac{(d_9 + \delta d_{11})}{(\theta_m - \sin(x + \gamma t))}, \quad (Nu)_{\eta=-1} = \frac{(d_8 + \delta d_{10})}{(\theta_m - 1)},$$

where $\theta_m = d_{14} + \delta d_{15}$.

The local rate of mass transfer coefficient (Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w} \left(\frac{\partial C}{\partial y} \right)_{y=\pm 1} \quad \text{where} \quad C_m = 0.5 \int_{-1}^1 C dy$$

and the corresponding expressions are

$$(Sh)_{\eta=+1} = \frac{(d_4 + \delta d_6)}{(C_m)}, \quad (Sh)_{\eta=-1} = \frac{(d_5 + \delta d_7)}{(C_m - 1)},$$

where $C_m = d_{12} + \delta d_{13}$

where d_1, d_2, \dots, d_{14} are constants.

5. DISCUSSION OF THE RESULTS

In this analysis, we investigate the effect of chemical reaction and radiation absorption on unsteady convective heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel in the presence of heat generating sources. The axial velocity is shown in figure 1 – 4 for different values of M , Q_1 , β and K . The actual axial flow is in the vertically downward direction i.e. $u < 0$ is actual flow and $u > 0$ is a reversal flow.

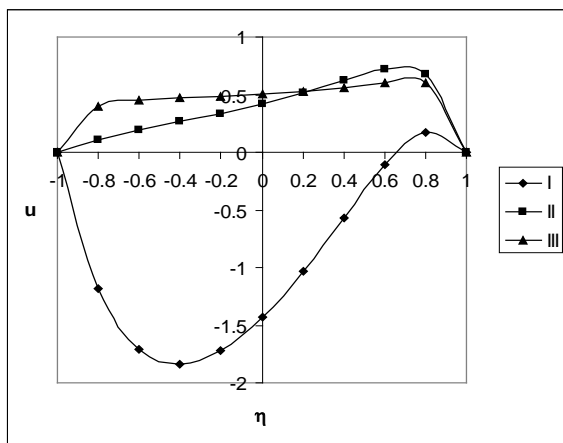


Fig. 1: Variation of u with M

	I	II	III
M	2	5	10

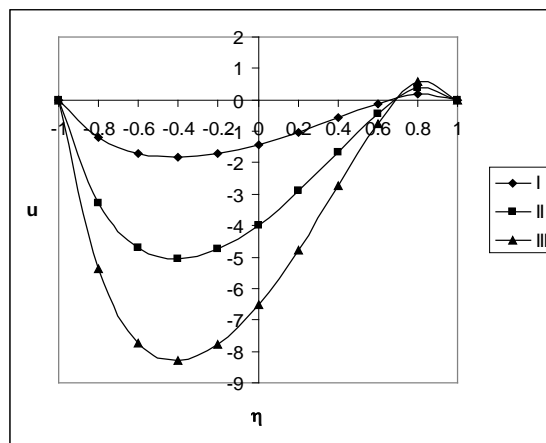


Fig. 2: Variation of u with Q_1

	I	II	III
Q_1	2	4	6

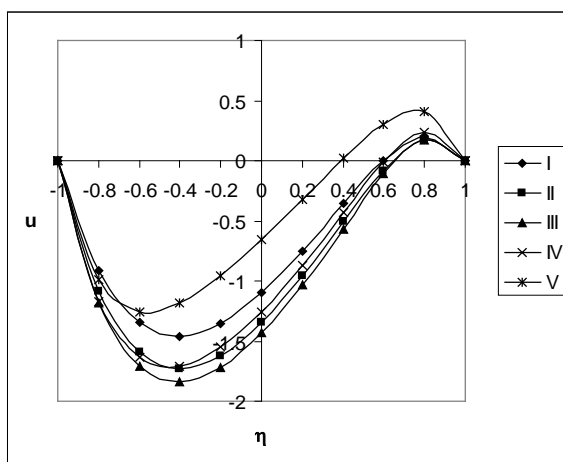


Fig. 3: Variation of u with β

	I	II	III	IV	V
β	0.1	0.3	0.5	0.7	0.9

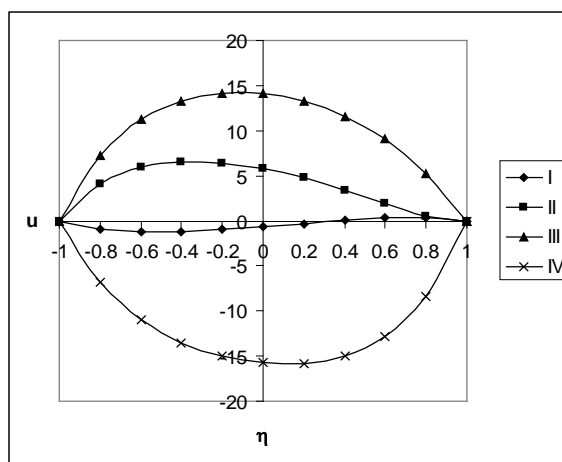


Fig. 4: Variation of u with k

	I	II	III	IV
k	1.5	2.5	3.5	4.5

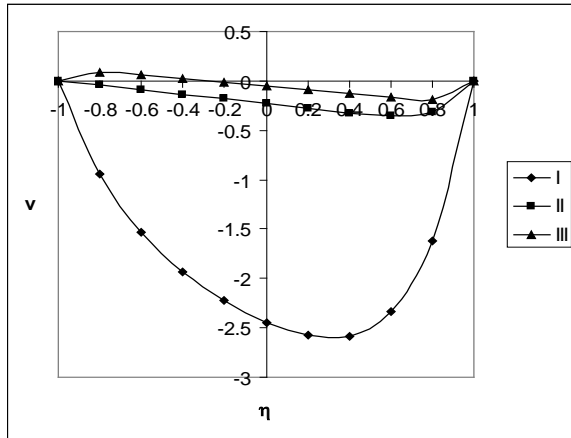


Fig. 5 : Variation of v with M

	I	II	III
M	2	5	10

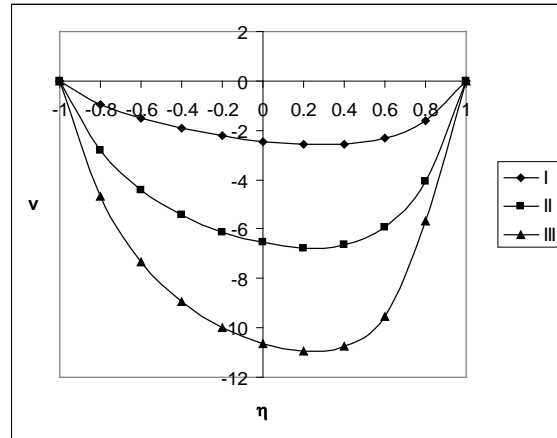


Fig. 6: Variation of v with Q_1

	I	II	III
Q_1	2	4	6

Higher the Lorentz force lesser $|u|$ in the flow region (fig.1). An increase in the radiation absorption parameter Q_1 results in an enhancement in $|u|$ with reversal flow in the vicinity of $\eta = 1$ (fig.2). It is found that higher the dilation of the channel walls, larger $|u|$ for $\beta \leq 0.5$ and larger for $|\beta| \geq 0.7$ (fig.3). The variation of u with chemical reaction parameter k shows that the axial velocity enhances with increase in k (fig.4).

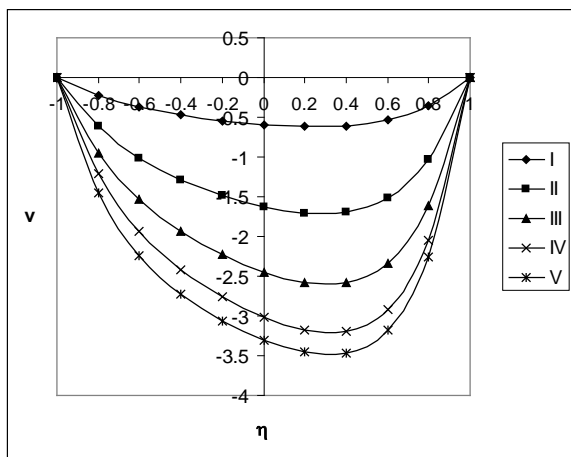


Fig. 7: Variation of v with β

	I	II	III	IV	V
β	0.1	0.3	0.5	0.7	0.9

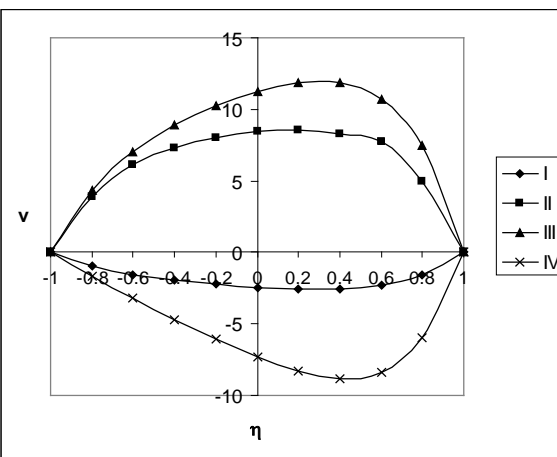


Fig. 8: Variation of v with k

	I	II	III	IV
k	1.5	2.5	3.5	4.5

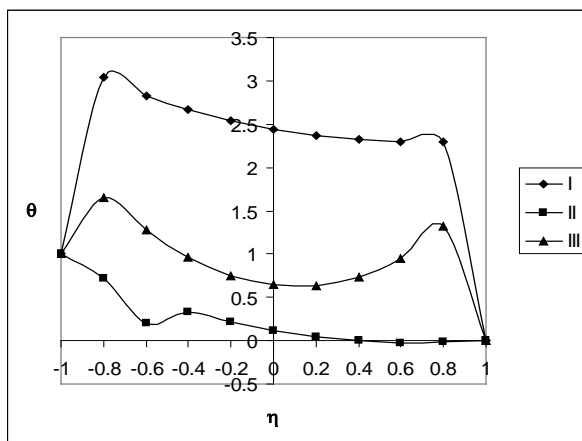


Fig. 9 : Variation of θ with M

	I	II	III
M	2	5	10

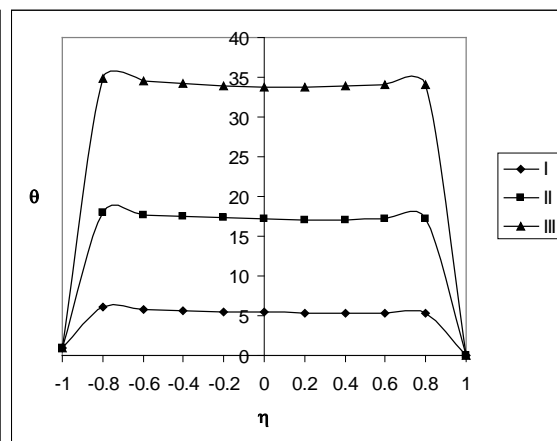


Fig. 10 : Variation of θ with Q_1

	I	II	III
Q_1	2	4	6

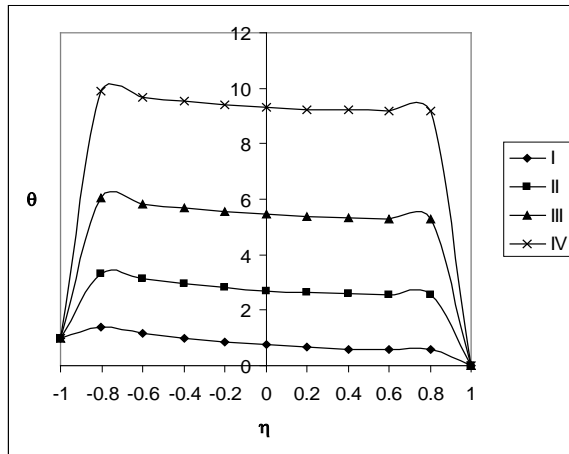


Fig. 11 : Variation of θ with β

	I	II	III	IV	V
β	0.1	0.3	0.5	0.7	0.9

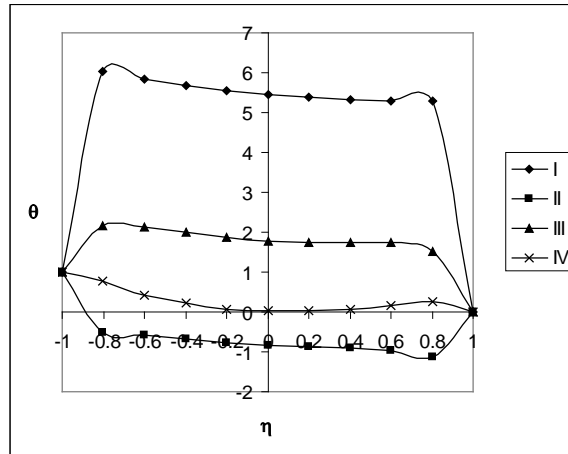


Fig. 12 : Variation of θ with k

	I	II	III	IV
k	1.5	2.5	3.5	4.5

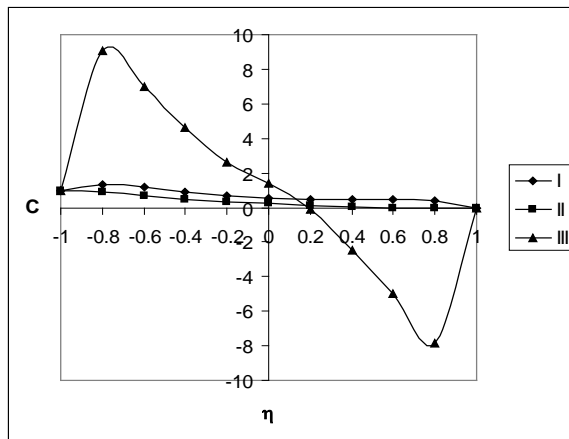


Fig. 13 : Variation of C with M

	I	II	III
M	2	5	10

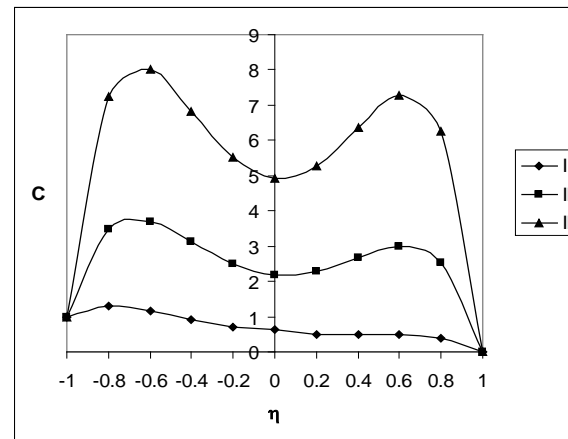


Fig. 14 : Variation of C with Q_1

	I	II	III
Q_1	2	4	6

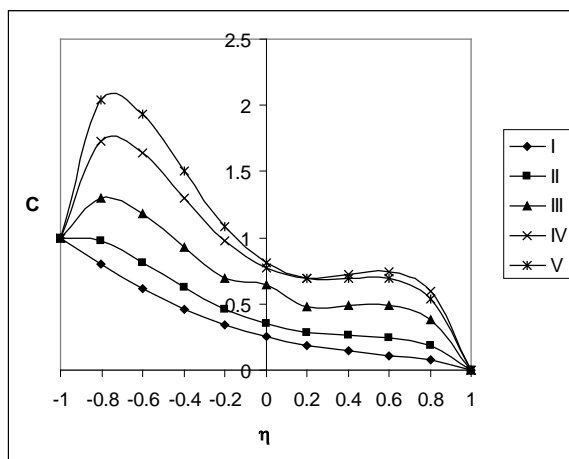


Fig. 15 : Variation of C with β

	I	II	III	IV	V
β	0.1	0.3	0.5	0.7	0.9

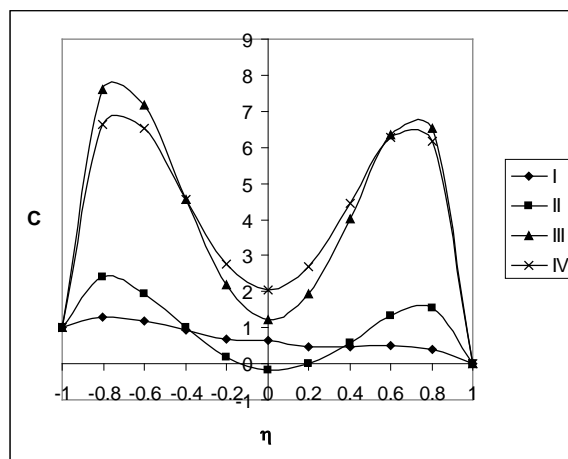


Fig. 16 : Variation of C with k

	I	II	III	IV
k	1.5	2.5	3.5	4.5

The secondary velocity (v) which is due to the wall waviness of the channel is shown in figs 5-8. Higher the Lorentz force smaller $|v|$ in the flow region. The depreciation in $|v|$ is remarkable at $M = 5$ and is marginal at higher $M=10$. (fig.5). An increase in Q_1 leads to an enhancement in $|v|$ (fig.6). From fig.7 we find that higher the dilation of the channel walls larger $|v|$ in the flow region. The variation of v with chemical reaction parameter k shows that $|v|$ enhances with increase in $k \leq 3.5$ and depreciates with higher $k \geq 4.5$ (fig.8).

The non-dimensional temperature (θ) is shown in figures 9-12 for different parametric values. We follow the convention that the non-dimensional temperature is positive or negative according as the actual temperature is greater (or) lesser than T_2 . The actual temperature depreciates with M (fig. 9). From fig 10, we notice that the actual temperature enhances with increase in the radiation absorption parameter Q_1 . The influence of the wall waviness is shown in (fig.11). It is found that higher the dilation of the channel walls larger the actual temperature. An increase in the chemical reaction parameter $k \leq 2.5$ leads to a depreciation in the actual temperature, enhances at $k = 3.5$ and again depreciates with higher $k \geq 4.5$ (fig.12).

The non-dimensional concentration (C) is shown in figs 13-16 for different parametric values. The variation of C with Hartmann number M shows that higher the Lorentz force smaller the actual temperature and for further higher Lorentz force larger in the left half and smaller in the right half (fig.13). An increase in Q_1 enhances in the entire flow region (fig.14). The influence of the wall waviness is shown in (fig.15). It is found that higher the dilation of the channel walls larger the actual concentration. For $k \leq 2.5$, we find that the actual concentration enhances in the region, except in the region $(-0.2, 0.2)$ where it depreciates and for higher $k \geq 3.5$, it depreciates in the regions adjacent to $\eta = \pm 1$ and enhances in the central region (fig.16).

The rate of heat transfer (Nu) at $\eta = \pm 1$ is shown tables 1-2 for different values of M , β , k , Q_1 . It is found that the rate of heat transfer enhances at $\eta = 1$ and reduces at $\eta = -1$ with $G > 0$ and for $G < 0$, we notice an enhancement at both the walls. An increase in M enhances $|Nu|$ at $\eta = \pm 1$. Higher the Lorentz force larger at $\eta = \pm 1$ while at $\eta = -1$, smaller $|Nu|$ and for further higher Lorentz force larger $|Nu|$. The variation of Nu with chemical reaction parameter k shows that $|Nu|$ enhances with k at $\eta = \pm 1$. An increase in Q_1 reduces it at $\eta = +1$ and enhances at $\eta = -1$.

The rate of mass transfer (Sh) at $\eta = \pm 1$ is shown in tables 3 – 4 for different parametric values. An increase in $G > 0$ enhances $|Sh|$ at $\eta = \pm 1$ while it enhances at $\eta = +1$ and reduces at $\eta = -1$ for $G < 0$. The rate of mass transfer enhances at $\eta = \pm 1$ with $M \leq 4$ and for higher $M \geq 6$, it enhances at $\eta = +1$ and reduces at $\eta = -1$. An increase in the radiation absorption parameter Q_1 enhances the rate of mass transfer at $\eta = +1$ and at $\eta = -1$, it reduces with $Q_1 \leq 4$ for $G > 0$ and enhances for $G < 0$ and for higher $Q_1 \geq 6$, it reduces for all G .

Table-1: Nusselt number (Nu) at $\eta = +1$

G	I	II	III	IV	V	VI	VII
10^3	-0.00337	-0.34727	-6.91830	-0.15009	-0.2512	-0.05328	-0.05940
3×10^3	-0.00698	-0.28757	-6.47917	-0.17771	-0.2715	0.01036	0.09965
5×10^3	-0.01264	-0.23384	-6.07054	-0.17728	-0.2911	0.05678	0.21915
-10^3	-0.00515	-0.41693	-7.39147	-0.06977	-0.0714	-1.17251	-0.36295
-3×10^3	-0.03287	-0.49922	-7.90280	0.11382	-0.1192	-0.89185	6.60215
-5×10^3	0.27118	-0.59787	-8.45711	0.51517	0.5314	0.36967	0.14976
M	2	4	6	2	2	2	2
k	0.5	0.5	0.5	1.5	2.5	0.5	0.5
Q_1	2	2	2	2	2	4	6

Table-2: Nusselt number (Nu) at $\eta = -1$

G	I	II	III	IV	V	VI	VII
10^3	0.30132	-0.08990	-9.02789	0.61643	0.71643	0.41680	0.50759
3×10^3	0.20156	-0.03521	-8.33573	0.39136	0.41138	0.32303	0.44518
5×10^3	0.14552	0.01004	-7.70740	0.26869	0.36769	0.29137	0.47656
-10^3	0.55672	-0.15728	-9.79415	1.05419	1.15324	0.78730	1.01501
-3×10^3	3.65776	-0.24227	-10.64706	2.02311	2.25113	-5.25261	-1.78571
-5×10^3	-0.72613	-0.35271	-11.60219	4.92682	5.45236	-0.60105	-0.66296
M	2	4	6	2	2	2	2
k	0.5	0.5	0.5	1.5	2.5	0.5	0.5
Q_1	2	2	2	2	2	4	6

Table-3: Sherwood number (Sh) at $\eta = +1$

G	I	II	III	IV	V	VI	VII
10^3	-0.00433	0.31460	0.37737	0.85633	-0.52111	1.06125	2.36756
3×10^3	0.22910	-0.07873	-2.46210	1.51080	0.55367	2.65128	6.41349
5×10^3	0.39619	-0.47268	-5.43847	1.95975	0.56229	4.77216	13.31212
-10^3	-0.29950	0.70687	3.08841	0.16688	-0.65218	-0.13264	-0.08798
-3×10^3	-0.65134	1.09766	5.67881	-0.30296	0.10005	-1.01993	-1.53827
-5×10^3	-1.05440	1.48654	8.15574	-0.46478	0.25704	-1.66209	-2.28564
M	2	4	6	2	2	2	2
k	0.5	0.5	0.5	1.5	2.5	0.5	0.5
Q_1	2	2	2	2	2	4	6

Table-4: Sherwood number (Sh) at $\eta = -1$

G	I	II	III	IV	V	VI	VII
10^3	-9.13616	-3.24148	-0.62710	-9.27785	-1.23367	-10.25605	-11.40274
3×10^3	-11.00023	-2.81517	1.66317	-15.05510	-2.36074	-9.56771	-8.12803
5×10^3	-13.31674	-2.39438	3.86735	1.49560	-2.49667	-7.99190	-3.36845
-10^3	-7.75220	-3.37388	-3.00914	-8.39664	-3.99941	-9.37967	-10.48924
-3×10^3	-6.90270	-4.11295	-5.48913	-8.75085	-3.08048	-5.65396	2.82965
-5×10^3	-6.68133	-4.55930	-8.07379	1.18119	-2.92544	3.60951	64.41862
M	2	4	6	2	2	2	2
k	0.5	0.5	0.5	1.5	2.5	0.5	0.5
Q_1	2	2	2	2	2	4	6

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