International Journal of Mathematical Archive-6(3), 2015, 95-105
Available online through www.ijma.info ISSN 2229 – 5046

PAIRWISE NON-ISOMORPHIC DECOMPOSITION OF GRAPHS

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(Received On: 10-03-15; Revised & Accepted On: 31-03-15)

ABSTRACT

Let G= (V, E) be a connected simple graph of order p and size q. If $G_1, G_2, ..., G_n$ are edge disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup ... \cup E(G_n)$,then $(G_1, G_2, ..., G_n)$ is said to be a decomposition of G. If each $G_i \cong H$ for some subgraph H of G, then $(G_1, G_2, ..., G_n)$ is said to be an isomorphic decomposition of G. Otherwise it is called a non-isomorphic decomposition. In this paper, we introduce pairwise non-isomorphic decomposition of graphs as a decomposition where G_i is not isomorphic to G_j for all $i \neq j$ and investigate graphs which admit such decomposition.

Keywords: *Decomposition, Pairwise non-isomorphic decomposition.*

AMS Subject Classification: *05C70.*

1. INTRODUCTION

By a graph, we mean a finite, undirected simple connected graph G without loops or multiple edges. The *degree* of any vertex u in G is the number of edges incident with *u* and is denoted by $d(u)$ and $\Delta(G)$ denotes the maximum degree of a graph. The *distance* between two vertices *u* and *v* of *G* is the length of the shortest *u-v* path in *G* and is denoted by d(u,v). The maximum distance between two vertices in a graph G is called the *diameter* of G and is denoted by diam(G). A *path* of length n is denoted by P_{n+1}. A *cycle* of length *n* is denoted by C_n. A connected acyclic graph is called a *tree*. A *complete graph* on n vertices is denoted by K_n. $W_n = C_n + K_1$ is called a *wheel*. $K_{1,n}$ denotes the *star graph*. K_n^+ denotes the graph obtained by identifying a pendent edge with every vertex of K_n. Terms not defined here are used in the sense of [7].

Let G = (V,E) be a connected simple graph of order *p* and size *q*. If $G_1, G_2, ..., G_n$ are edge disjoint sub graphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup ... \cup E(G_n)$ then $(G_1, G_2, ..., G_n)$ is said to be a *decomposition* of G. If each $G_i \cong H$ for some subgraph *H* of *G*, then $(G_1, G_2, ..., G_n)$ is said to be an *isomorphic decomposition* of *G*. Otherwise it is called a *non-isomorphic decomposition*. Different types of decomposition of G have been studied in literature by imposing suitable conditions on the subgraphs G_i. Isomorphic decompositions are found in [5], [6], [11] and [12] and non-isomorphic decompositions are dealt in [1], [2], [3], [4], [8], [9], [13] and [14].

In this paper, we introduce the concept of pairwise non-isomorphic decomposition of graphs and investigate standard graphs which admit such decomposition. We also get bounds for diameter and maximum degree for certain graphs which admit such decompositions.

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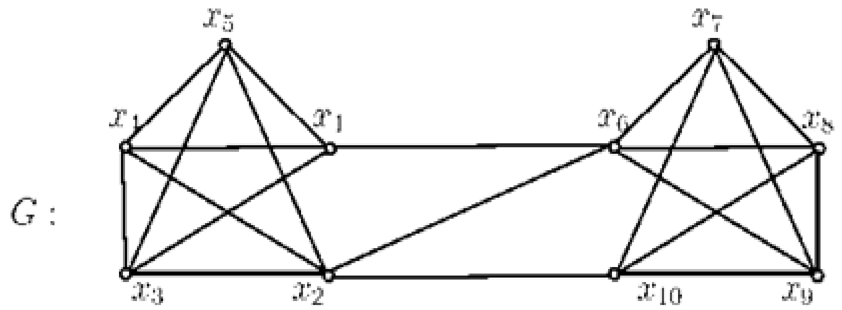
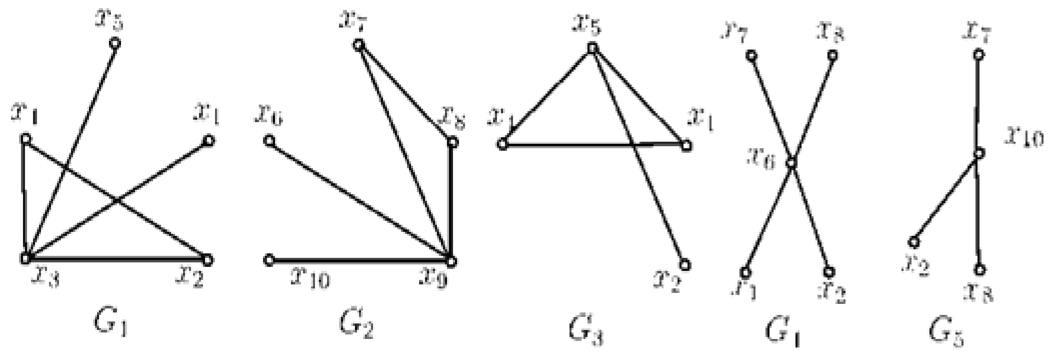
2. DEFINITIONS AND EXAMPLES

Definition 2.1: A decomposition $(G_1, G_2, ..., G_n)$ of G is said to be pairwise non-isomorphic decomposition (PND) if G_i is not isomorphic to G_j for all $i \neq j$.

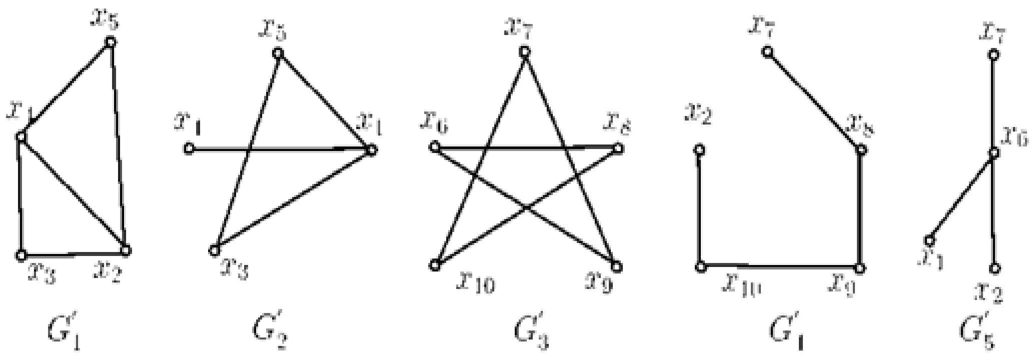
In non- isomorphic decomposition, two subgraphs may be isomorphic, but it is not allowed in PND. For the graph G given in figure 2.1, $(G_1, G_2, G_3, G_4, G_5)$ is a non-isomorphic decomposition and $(G_1', G_2', G_3', G_4', G_5')$ is a PND of G.



(a) A connected graph G



(b) A non-isomorphic decomposition of G.



(c) A Pairwise non –isomorphic decomposition of G.

Fig 2.1: *G* and its decompositions

Since graphs of different sizes are obviously non-isomorphic, we concentrate on the decomposition of a connected graph into pairwise non-isomorphic connected subgraphs of a particular size. The non-isomorphic connected graphs of size 4 are given in Figure 2.3.

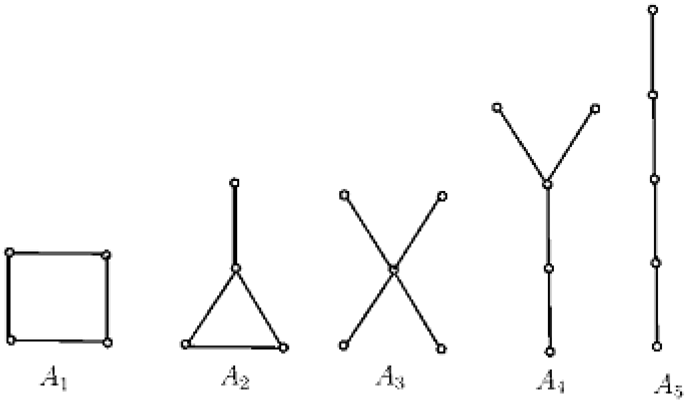


Figure 2.3: Non-isomorphic connected graphs of size 4

Example 2.2: The connected graph G given in Figure 2.4 can be decomposed into pairwise non-isomorphic connected subgraphs of size 4 where

$A_1 : \langle x_1x_2, x_2x_8, x_8x_7, x_7x_1 \rangle$, $A_2 : \langle x_1x_4, x_4x_2, x_2x_3, x_3x_4 \rangle$, $A_3 : \langle x_5x_6, x_5x_1, x_5x_2, x_5x_3 \rangle$, $A_4 : \langle x_8x_9, x_9x_3, x_9x_{10}, x_3x_1 \rangle$, $A_5 : \langle x_7x_6, x_6x_{10}, x_{10}x_4, x_4x_5 \rangle$.

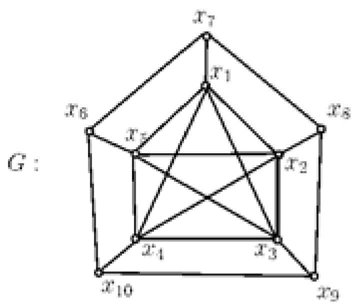


Figure 2.4

Definition 2.3: A PND is said to be *full pairwise non-isomorphic decomposition (FPND)* if the decomposition contains all possible subgraphs of particular size.

Remark 2.4: If a graph G contains neither C_3 nor C_4, then G does not admit a FPND into subgraphs of size 4, but not conversely. That is, if G contains C_3 and C_4, then G need not admit FPND.

For example, the graph given in Figure 2.5 contains both C_3 and C_4, but it does not admit FPND into subgraphs of size 4.

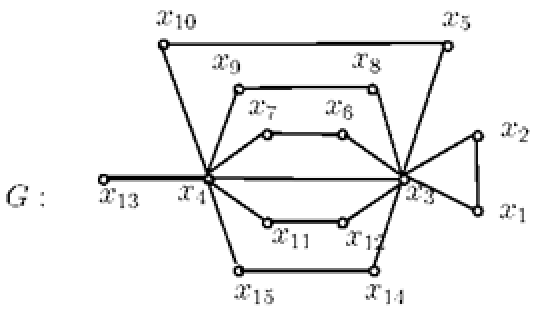


Figure 2.5

Since $G - x_3x_4$ contains no C_4, without loss of generality, let $A_1 = \langle x_3, x_4, x_6, x_7 \rangle$. Since the graph G contains exactly one C_3, let $A_2 = \langle x_1, x_2, x_3, x_5 \rangle$.

Now, $G-E(A_1)\cup E(A_2)$ has exactly one vertex x_4 of degree at least 4. Hence we let $A_3=\langle x_4,x_9,x_{11},x_{13},x_{15}\rangle$. Now $G-E(A_1)\cup E(A_2)\cup E(A_3)$ has exactly one vertex x_3 of degree 3. Hence we let $A_4=\langle x_3,x_8,x_{12},x_{14},x_9\rangle$.

Then $G-E(A_1)\cup E(A_2)\cup E(A_3)\cup E(A_4)$ is a disconnected graph and it contains K_2 as a component. Hence G does not admit a FPND into connected subgraphs of size 4.

Notation 2.5: The pairwise non-isomorphic decomposition of G into *l*- subgraphs, each of size *k* is denoted by *(k,l)-PND.* Then it is necessary that $|E|=lk$.

Example 2.6: For the graph G given in Figure 2.6, (G_1,G_2,G_3) is a (4, 3) - PND.

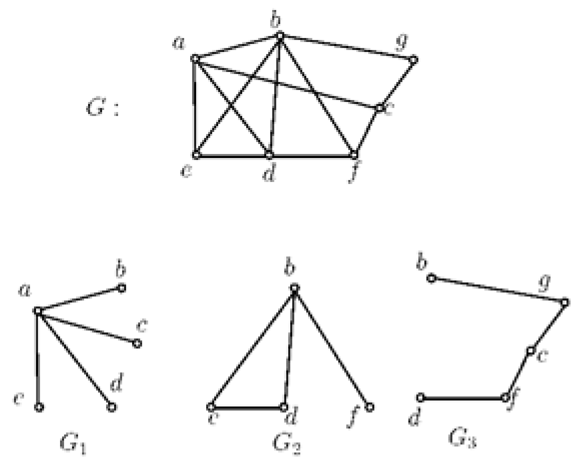
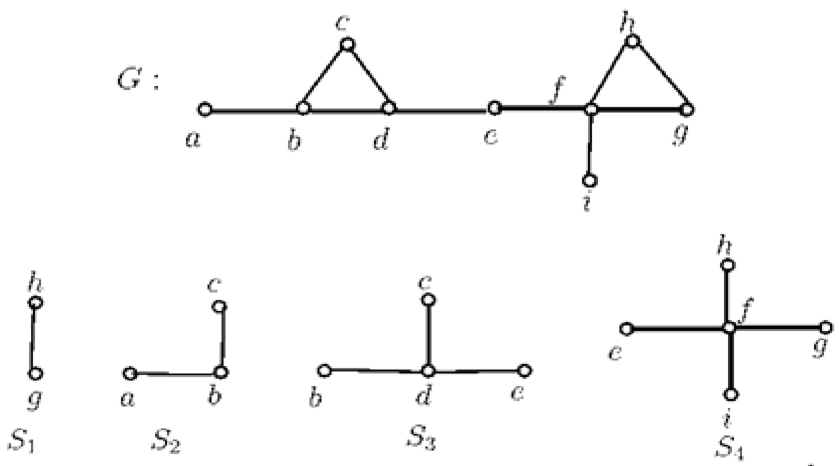


Figure 2.6

Definition 2.7: In [8], Gnanadhas and Paulraj Joseph introduced continuous monotonic decomposition of graphs and investigated various properties of the decomposition. A decomposition $(G_1,G_2,...,G_n)$ of G is said to be a *continuous monotonic decomposition* (CMD) if each G_i is connected and $|E(G_i)|=i$ for each *i*=1, 2,….n.

Definition 2.8: [8] A CMD in which each G_i is a star is said to be *continuous monotonic star decomposition (CMSD).*

Example 2.9: For the graph G in Figure 2.7, (S_1,S_2,S_3,S_4) is a CMSD of the graph G. We note that every CMSD is a PND but not conversely.



A CMSD of G
Figure 2.7

3. PND FOR STANDARD GRAPHS

In this section, we investigate the pairwise non-isomorphic decomposition of some standard graphs.

Since a subgraph of a star is also a star, a star of size $n=lk$ cannot be decomposed into pairwise non-isomorphic substars of size k. Similarly any subgraph of a path or a cycle is a path and hence cycles and paths of size $n=lk$ do not admit a PND of size k. Thus $P_n, K_{1,n}$ and C_n do not admit PND.

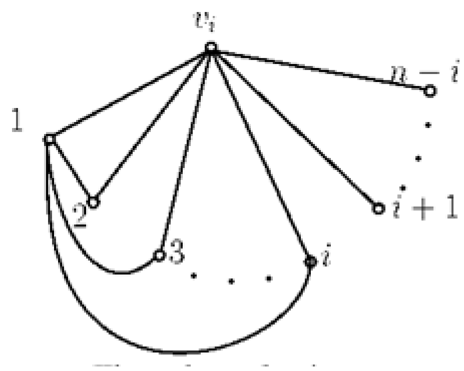
Since $E(W_n) = E(C_n) \cup E(K_{1,n})$, W_n is decomposed into C_n and $K_{1,n}$ and hence it admits (n,2)- PND.

Theorem 3.1: For $n \geq 4, K_n$ *admits a pairwise non-isomorphic decomposition.*

Proof: Let $V(K_n) = \{v_1, v_2, ...v_n\}$. Now $\left|E(K_n)\right| = \dfrac{n(n-1)}{2}$.

Let $G_n = K_{1,n-1}$ be a subgraph of G obtained by taking v_1 as the center vertex of the star and $G_n' = G - \{v_1\}$. Similarly let $G_{n-1} = K_{1,n-2}$ be a subgraph of G_n' obtained by taking v_2 as the center vertex of the star and $G_{n-1}' = G_n' - \{v_2\}$. Proceeding like this, finally we get $G_2 = K_{1,1} = \langle v_{n-1}, v_n \rangle$, which is a subgraph of $G_3' = G_4' - \{v_{n-2}\}$. Now $G_2, G_3, ..., G_n$ form a CMSD of K_n.

If n is even, then take $A_1 = G_n$ and $A_i = \langle E(G_{n+1-i}) \bigcup E(G_i) \rangle, i = 2,3,..., \dfrac{n}{2}$. Then for each i, A_i is isomorphic to the subgraph of size n-1 given in Figure 3.1.



The subgraph A_i

Figure 3.1

Clearly A_i' s form an (n-1, n/2)- PND.

If n is odd, then take $A_i = \langle E(G_{n+1-i}) \bigcup E(G_{i+1}) \rangle, i = 1,2,3,..., \dfrac{n-1}{2}$. Then for each i, A_i is isomorphic to the subgraph of size n and these $\dfrac{n-1}{2}$ connected pairwise non-isomorphic subgraphs of size n give PND of K_n.

Theorem 3.2: For $n \geq 3, K_n^{\;+}$ *admits a pairwise non-isomorphic decomposition.*

Proof: Let $V(K_n^{\;+}) = \{v_1, v_2, ...v_n\} \cup \{u_1, u_2, ..., u_n\}$.

Without loss of generality, we may assume that $\{u_1, u_2, ..., u_n\}$ is the set of all end vertices of K_n^+ such that u_i is adjacent to v_i. Now $\left|E(K_n^+)\right| = \frac{n(n+1)}{2}$.

Let $G_n = K_{1,n}$ be a subgraph of G obtained by taking v_1 as the center of the star and $G_n' = G - \{v_1\}$. Similarly, let $G_{n-1} = K_{1,n-1}$ be a subgraph of G obtained by taking v_2 as the center of the star and $G_{n-1}' = G_n' - \{v_2\}$. Proceeding like this, finally we get $G_1 = K_{1,1} = \langle v_n u_n \rangle$, which is a subgraph of $G_2' = G_3' - \{v_{n-1}\}$. Now $G_1, G_2, ..., G_n$ form a CMSD of K_n^+. If n is even, then take $A_i = \langle E(G_{n+1-i}) \cup E(G_i) \rangle, i = 1, 2, ..., \frac{n}{2}$.

For each i, A_i is isomorphic to the subgraph of size n+1, given in Figure 3.2.

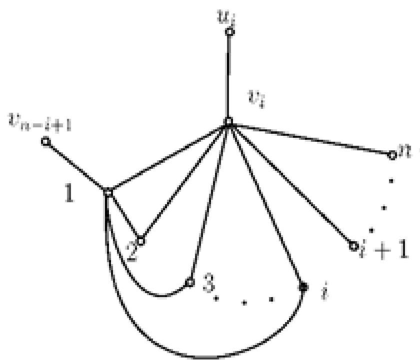


Figure 3.2: The subgraph A_i

Clearly the A_i's are the $\frac{n}{2}$ connected pairwise non-isomorphic subgraphs of size n+1. If *n* is odd, then take $A_1 = G_n$ and $A_i = \langle E(G_{n-i+1}) \cup E(G_{i-1}) \rangle, i = 2, 3, ..., \frac{n+1}{2}$.

Then for each, i, A_i is isomorphic to the subgraph of size *n* and these $\frac{n+1}{2}$ connected pairwise non-isomorphic subgraphs of size *n* give a PND of K_n^+.

4. BOUNDS FOR SOME GRAPH PARAMETERS

In this section, we obtain bounds for the diameter and maximum degree of graphs which admit *(k,l)*-PND. We prove similar results for other parameters for some special graphs.

Theorem.4.1: *If a graph G admits (k,l)-PND ,* $k \geq 4$ *and* $l \geq \left\lfloor \frac{k(k+4)}{4} - 3 \right\rfloor$, *then*

$$diam(G) \leq \left\lfloor l(k-3) + \frac{k^2 + 6k - 7}{4} \right\rfloor.$$

Proof: Since we need the upper bound for diam(G), we start with paths of length k,k-1,…and so on. P_{k+1} is the only graph with size k which contribute *k* edges to diam (G).

If a subgraph contributes k-1 edges to diam(G), then the remaining one edge may be incident with any one of the internal vertices of P_k. Thus there are exactly $\left\lfloor \frac{k-1}{2} \right\rfloor$ non-isomorphic graphs of size k.

If a subgraph contributes k-2 edges of diam(G), then the remaining two edges of the graph are identified at any vertex of P_{k-1} without affecting the diameter. If one end of P_3 is identified at any one of the internal vertices of P_{k-1}, then there are exactly $\left\lfloor \frac{k-2}{2} - 1 \right\rfloor$ non-isomorphic graphs of size k. If the two ends of P_3 are identified at any two adjacent vertices of P_{k-1}, then there are exactly $\left\lceil \frac{k-2}{2} \right\rceil$ non-isomorphic graphs of size k. If the two ends of P_3 are identified at any two vertices of $P_{k-1,}$ which are at a distance two in $P_{k-1,}$ then there are $\left\lfloor \frac{k-2}{2} \right\rfloor$ non-isomorphic graphs of size k. If the internal vertex of P_3 is identified at any one of the internal vertices of P_{k-1}, then there are exactly $\left\lfloor \frac{k-2}{2} \right\rfloor$non-isomorphic graphs of size k. If two pendent edges are identified at two different internal vertices of P_{k-1} then there are (k-4) + (k-6) + …+4+2 (if k is even) or (k-4) + (k-6) + …+ 3+ 1 (if k is odd) non-isomorphic graphs of size k.

If k is even, then

$$\text{diam(G)} = k + (k-1)\left(\frac{k-2}{2}\right) + (k-2)\left[\left(\frac{k-4}{2}\right) + \left(\frac{k-2}{2}\right) + \left(\frac{k-2}{2}\right) + \left(\frac{k-2}{2}\right) + (k-4) + (k-6) + \ldots + 4 + 2\right]$$

+ (k-3) [number of graphs of size k which contribute k-3 edges to the diameter] + (k-4) [Number of graphs of size k which contribute k- 4 edges to the diameter] + …

$$\leq k + (k-1)\left(\frac{k-2}{2}\right) + (k-2)\left[\left(\frac{4k-10}{2}\right) + \left(\frac{k-4}{2}\right)\left(\frac{k-2}{2}\right)\right] + (k-3)$$

[Number of graphs of size *k* which contribute at most *(k-3)* edges to the diameter].

Since $l = 1 + \frac{k-2}{2} + \left[\frac{k-2}{2} + \frac{k-4}{2} + \frac{k-2}{2} + \frac{k-2}{2} + (k-4) + (k-6) + \ldots + 4 + 2\right]$ + [Number of graphs of size *k* which contribute *(k-3)* edges to the diameter]+[Number of graphs of size *k* which contribute *(k-4)* edges to the diameter]+ …, the number of graphs of size *k* which contribute at most (*k*-3) lengths to the diameter is

$$l - \left(1 + \frac{k-2}{2} + \frac{k-2}{2} + \frac{k-4}{2} + \frac{k-2}{2} + \frac{k-2}{2} + 2\left[1 + 2 + \ldots + \left(\frac{k-4}{2}\right)\right]\right)$$

$$= l - \left(\frac{k(k+4)}{4} - 3\right)$$

By hypothesis $l \geq \left\lfloor \frac{k(k+4)}{4} - 3 \right\rfloor$,

$$\text{diam(G)} \leq k + (k-1)\left(\frac{k-2}{2}\right) + (k-2)\left[\left(\frac{4k-10}{2}\right) + \left(\frac{k-4}{2}\right)\left(\frac{k-2}{2}\right)\right] + (k-3)\left[l - \left(\frac{k(k+4)}{4} - 3\right)\right]$$

$$= l(k-3) + \frac{k^2+6k-8}{4}, \text{ after simplification}$$

$$= \left\lfloor l(k-3) + \frac{k^2+6k-7}{4} \right\rfloor.$$

Similarly, we can prove that if *k* is odd, $diam(G) \leq l(k-3) + \frac{k^2+6k-7}{4}$. Thus if $l \geq \left\lfloor \frac{k(k+4)}{4} - 3 \right\rfloor$, then we have $diam(G) \leq \left\lfloor l(k-3) + \frac{k^2+6k-7}{4} \right\rfloor$. Hence the result.

Theorem 4.2: *If a graph G admits (k,l)-PND with* $k \geq 6$*, then* $\Delta(G) \leq l(k-3)+15.$

Proof: Let $d(v) = \Delta(G).$

Since we need the upper bound for $\Delta(G)$, we start with stars of size k,k-1,… and so on. There is only one subgraph $K_{1,k}$ which contributes *k* edges to d(v). If a subgraph contributes k-1 edges to d(v), then the remaining one edge may be incident with any one of the end vertices of $K_{1,k-1}$ (or) the ends of the edge may be identified with any two ends of $K_{1,k-1}$. In this case we get exactly two graphs of size *k.*

If a subgraph contributes *k-2* edges to d(v), then the remaining two edges may be incident with any end vertex of $K_{1,k-2}$ in the following eight types: one end of P_3 is identified with any one end of $K_{1,k-2}$ (or) the internal vertex of P_3 may be identified with any one end vertex of $K_{1,k-2}$ (or) all the three vertices of P_3 are identified with any three end vertices of $K_{1,k-2}$ (or) any two adjacent vertices of P_3 are identified with any two end vertices of $K_{1,k-2}$ (or) the two ends of P_3 are identified with any two end vertices of $K_{1,k-2}$ (or) two pendent edges are incident with two pendent vertices of $K_{1,k-2}$ (or) one pendent edge is incident with any one pendent vertex of $K_{1,k-2}$ and the two ends of another edge may be identified with two different end vertices of $K_{1,k-2}$ (or) both ends of two edges are identified with different end vertices of $K_{1,k-2}$. Proceeding like this we get,

$\Delta(G) = k + 2(k-1) + 8(k-2) + (k-3)$ [Number of graphs of size k which contribute k-3 edges to d(v)]+(k-4) [Number of graphs of size k which contributes k-4 edges to d(v)]+ ….

$\leq 11k - 18 + (k-3)$ [Number of graphs of size k which contribute at most k-3 edges to d(v)]

Since *l*=1+2+8+[Number of graphs of size k which contribute at most k-3 edges to d(v)], *l*-11 is the number of graphs of size k which contribute at most k-3 edges to d(v) .

Thus $\Delta(G) \leq 11k - 18 + (k-3)(l-11) = l(k-3)+15$.

Corollary 4.3: *If a graph G admits (k,l)-PND of trees ,with* $k \geq 6$*, then* $\Delta(G) \leq l(k-3)+8.$

Corollary 4.4: *Let G be any graph obtained by attaching n independent vertices to any vertex v of a connected graph H. If G admits (k,l)- PND, then* $n \leq l(k-4)+9.$

Theorem 4.5: *If a graph with an induced subgraph* C_n *admits (k,l)- PND of trees and* $l \geq \left\lfloor \frac{(k-2)(k+2)}{4} \right\rfloor$*, then*

$n \leq \left\lfloor l(k-3) + \frac{k(k+2)+1}{4} \right\rfloor.$

Proof: Since we need the maximum value for n, we start with paths of length k, k-1,… and so on. P_{k+1} is the only tree with size *k* which contribute *k* edges to n. If a tree contributes *k*-1 edges to *n*, then the remaining one edge may be incident with any one of the internal vertices of P_k .Thus there are exactly $\left\lfloor \frac{k-1}{2} \right\rfloor$ non-isomorphic tress of size *k.*

If a tree contributes *k-2* edges to *n*, then the remaining two edges of the tree are identified at any internal vertices of P_{k-1} without affecting the non-isomorphism of trees. If one end of P_3 is identified at any one of the internal vertices of P_{k-1}, then there are exactly $\left\lfloor \frac{k-2}{2} - 1 \right\rfloor$ non-isomorphic trees of size k. If the internal vertex of P_3 is identified at any one of the internal vertices of P_{k-1}, then there are exactly $\left\lfloor \frac{k-2}{2} \right\rfloor$ non-isomorphic trees of size *k*. If two pendent edges are identified at two different internal vertices of P_{k-1}, then there are (k-4) + (k-6) +…+ 4 + 2 (if k is even) or (k-4) + (k-6) + …+ 3+1 (if *k* is odd) non-isomorphic trees of size *k*.

If *k* is even, then

$n = k + (k-1)\left(\frac{k-2}{2}\right) + (k-2)\left[\frac{k-4}{2} + \frac{k-2}{2} + (k-4) + (k-6) + \ldots + 4 + 2\right] + (k-3)$ [Number of trees of size *k* which contribute k-3 edges to n]+ (k-4) [Number of trees of size k which contribute k-4 edges to n] +…

$\leq k+(k-1)\left(\frac{k-2}{2}\right)+(k-2)\left[\frac{2k-6}{2}+\left(\frac{k-4}{2}\right)\left(\frac{k-2}{2}\right)\right]+(k-3)$ [Number of trees of size *k* which contribute at most *k-3* edges to n].

Since $l=1+\left(\frac{k-2}{2}\right)+\left[\frac{k-4}{2}+\frac{k-2}{2}+(k-4)+(k-6)+...+4+2\right]+$ [Number of trees of size *k* which contribute k-3 edges to n] + [Number of trees of size *k* which contribute k-4 edges to n] +…., the number of trees of size *k* which contribute at most k-3 edges to n is

$$l-\left(1+\left(\frac{k-2}{2}\right)+\left(\frac{k-4}{2}\right)+\left(\frac{k-2}{2}\right)+2\left(1+2+...+\left(\frac{k-4}{2}\right)\right)\right)=l-\left(\frac{(k+2)(k-2)}{4}\right)$$

By hypothesis $l\geq\left\lfloor\frac{(k+2)(k-2)}{4}\right\rfloor$

$$n\leq k+(k-1)\left(\frac{k-2}{2}\right)+(k-2)\left[\frac{2k-6}{2}+\left(\frac{k-4}{2}\right)\left(\frac{k-2}{2}\right)\right]+(k-3)\left(l-\left(\frac{(k+2)(k-2)}{4}\right)\right)$$

$n\leq l(k-3)+\frac{k(k+2)}{4}$, after simplification

Similarly, we can prove that if *k* is odd, then $n\leq l(k-3)+\frac{k(k+2)+1}{4}$. Thus if $l\geq\left\lfloor\frac{(k-2)(k+2)}{4}\right\rfloor$, *then* $n\leq\left\lfloor l(k-3)+\frac{k(k+2)+1}{4}\right\rfloor$. Hence the required result.

Definition 4.6: A path in which each edge is a bridge is said to be a path of bridges. In addition if each internal vertex is of degree 2, then it is called a simple path of bridges.

Example 4.7: For the graph *G* in Figure 4.1, $P: v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}$ is a path of bridges. Since $d(v_8)=4$ and $d(v_9)=3$, P is not a simple path of bridges. But $P': v_4, v_5, v_6, v_7, v_8$ is a simple path of bridges.

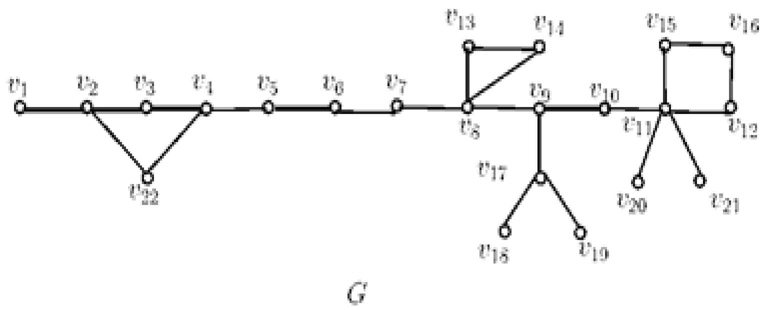


Figure 4.1

Theorem 4.8: *If a graph G admits a PND of size k, then the maximum length of simple path of bridges is 3k-5.*

Proof: Let P_{n+1} be any simple path of bridges of length n and $\{v_1, v_2,..., v_{n-1}\}$ be the set of all internal vertices of P_{n+1} and $d_G(v_i)=2, 1\leq i\leq n-1$. Then $G-\{v_1, v_2, ...v_{n-1}\}=G_1\cup G_2$.

Since G admits PND, G_1 and G_2 both can not be trivial. Clearly, $|V(G_i)\cap V(P_{n+1})|=1$ and let $u_i=V(G_i)\cap V(P_{n+1}), i=1,2$.

Case-(i): Any one of G_1 and G_2 is trivial.

Without loss of generality, we may assume that $G_1 \cong K_1 = \{u_1\}$ and G_2 is non-trivial. Thus $d_G(u_2) \geq 3$ and exactly one member of the decomposition may contain edges from both G_2 and P_{n+1}. Since we need the maximum value for n and a sub graph of a path is a path, we can choose a path P_{k+1} as a member without taking any edge of G_2. If a member contains k-1 edges in P_{n+1} and one edge in G_2, then the member must be P_{k+1}. But P_{k+1} is already chosen.

If a member contains *k-2* edges in P_{n+1} and two edges in G_2, then the required graph obtained by identifying the end vertices of two edges in G_2 with u_2. Thus there is exactly one member which contribute exactly $k-2$ edges to P_{n+1}. Thus in this case, we have $n \leq k + k - 2 = 2(k-1)$.

Case-(ii): Both G_1 and G_2 are non-trivial.

Then $d_G(u_i) \geq 3, i = 1,2.$ As discussed above, we can choose a path P_{k+1} as a member without taking any edge of G_1 and G_2. Thus there is no subgraph containing k-1 edges in P_{n+1} and one edge in G_i, (i =1 or 2). Also we can choose a member which contribute k-2 edges to P_{n+1} and two edges to G_i, (i = 1 or 2).

Now we claim that at least one of G_1 and G_2 contains at least three edges. Suppose $|E(G_i)| = 2, i = 1,2$. Then there is exactly one member which contribute exactly k-2 edges to P_{n+1}, which is a contradiction. Without loss of generality, we may assume that $|E(G_1)| \geq 3$ and $|E(G_2)| \geq 2.$ If a member contains k-3 edges in P_{n+1}, then three copies of P_2 are identified at the end vertex of P_{k-2} (or) the ends of one copy of P_3 and P_2 are identified at the end vertex of P_{k-2}(or) $C_3$is identified at the end vertex of P_{k-2}.

Thus $n \leq k + (k-2) + (k-3) = 3k - 5$

5. CONCLUSIONS

In this paper, we have introduced the concept of pairwise non-isomorphic decomposition of graphs and investigated standard graphs which admit such decompositions. We also got bounds for diameter and maximum degree for certain graphs. A specialized study of this concept for trees will be reported shortly.

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Source of support: Nil, Conflict of interest: None Declared