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FUZZY CRITICAL PATH BASED ON INTERVAL VALUED LINGUISTIC FUZZY SETS

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ABSTRACT

Fuzzy set theory plays an important role to aid research in fuzzy critical path analysis when the dynamics of the activity times are provided by fuzzy numbers. Previous studies on critical path analysis have demonstrated the use of fuzzy numbers as activity times in a fuzzy project network. In this paper, interval valued linguistic fuzzy sets are used to find fuzzy critical path in a fuzzy project network. The interval valued linguistic fuzzy variables can provide a more accurate activity times in project network analysis. New arithmetic operations on linguistic fuzzy numbers are defined to get accurate fuzzy critical path in a fuzzy project network using Technique for order preference by similarity to ideal solution (TOPSIS) method.

Keywords: *Interval valued fuzzy number, TOPSIS, linguistic fuzzy number, critical path, project network.*

1. INTRODUCTION

The theory of interval-valued fuzzy sets has been brought into light during the mid-seventies by Grattan-Guinness [10]. The theory states that the fuzzy membership surveyed upon interval and many-valued quantities. The perception about linguistic variables and its relevance to estimated reasoning has been introduced by L. Zadeh [24]. Interval valued fuzzy set theory is a progressively more widespread extension of the fuzzy set theory where the conventionally valued membership degrees [0, 1] are substituted by hiatus in [0, 1] that verge on the exact degrees that are partially unknown. Thus not just the ambiguity but also a trait of indecision can be dealt with spontaneously. Likewise, interval-valued fuzzy sets (IVFSs) are significantly easier to tackle in praxis than the correspondingly inspired type-2 fuzzy sets (of which IVFSs are, in fact, a special case), that are known as "interval type-2 fuzzy sets" in that milieu. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is an accepted methodology to Multiple Criteria Decision Making (MCDM) glitches. This has been put forth by Hwang and Yoon [11]. This modus has been extensively put into use in the literature (Abosinna and Amer [1], Agrawal and others [2] alongside Chen and Tzeng [4]). TOPSIS is a manifold criteria method to ascertain a solution from a set of predetermined replacements. The rudimentary principle is that the selected alternative should have the shortest distance from the positive idyllic solution and the farthest distance from the negative ideal solution. In the TOPSIS, the ratings of performance and the weights of the criteria are attributed as crisp values. In the midst of many a case, crisp data are insufficient to model real life situations. Chen [3] encompasses the method of TOPSIS to fuzzy group decision circumstance by cogitating triangular fuzzy numbers and circumscribing crisp Euclidean expanse between two fuzzy numbers. Tsaur and others [17] transfigured fuzzy MCDM hindrance into a crisp one by way of centroid defuzzification and then elucidated the non-fuzzy MCDM restraint using the TOPSIS approach. Opricovic with Tzeng [14] accompanied a comparative analysis of VIKOR and TOPSIS. The VIKOR (ViseKriterijumska Optimizacija I Kompromisno Resenje) procedure, expounded by Opricovic [15], is a compromise approach of ranking. It establishes a compromise solution, bestowing a thoroughgoing utility for the majority and a minutes regret for the opponent. It also brings in the multi-criteria index of ranking constructed on the particular measure of proximity to the ideal solution. Chu and Lin [7] transformed fuzzy MCDM setback into a crisp one. In contrast to the others, they first draw on the membership functions of all the weighted ratings in a weighted normalized decision matrix and then by the process of defuzzifying, transform them to crisp values. Jahanshahloo and others [12] refurbished an algorithmic procedure to outspread TOPSIS for decision-making hindrances with interval data. Yang and Hung [22] investigated the use of TOPSIS and fuzzy TOPSIS in deciphering a plant layout design drawback. The variance between TOPSIS and fuzzy TOPSIS is predominantly positioned in the approaches of rating. The redeeming feature of fuzzy TOPSIS is to ascribe the consequence of

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attributes and the performance of replacements with regard to innumerable characteristics by putting into use fuzzy numbers in place of precise numbers. There are essential steps in making use of TOPSIS implicating numerical measures of the comparative magnitude of attributes as well as the performance of each substitution on these attributes. Nevertheless, it may get very difficult to determine the exact data given that human inferences are frequently indistinguishable under many stipulations. Consequently, an augmentation `of TOPSIS to the fuzzy environs is a natural generalization of TOPSIS models by Chen, S.M. and Lee, L.W [6], G.R. Jahanshahloo and *et al.* [13]. Triantaphyllou and Lin [18] have fostered a fuzzy version of TOPSIS process built on fuzzy arithmetic strategies, which escort to a fuzzy relative imminence for each alternative. Except for Wang and Elhag [19], our investigation displays that the vital limitations in the other cited approaches are the loss of material (defuzzification) in the preliminary phases of their procedure. At times, the available information is not adequate for the scrupulous delineation of a degree of membership for certain rudiments. There may be some vacillating degree between membership and non-membership. Taking into consideration that there are number of real life instances where there is an inadequacy in information availability, it can be derived that Interval-Valued Fuzzy Sets (IVFSs) with ill-equipped membership grades are apposite to deal with such problems. The system of IVFS is delimited by an interval-valued membership function by L. Zadeh [24] and R. Sambuc, Fonctions-floues [16]. In other words, the degree of membership of a component to a set is portrayed by a cliquish subinterval of [0, 1]. IVFSs have been found to be remarkably constructive to apportion with vagueness. Wang and Elhag's approach of fuzzy TOPSIS is construed upon alpha level sets and the fuzzy extension principle, which works out the fuzzy relative proximity of each alternative by disentangling the non-linear programming models. Final ranking is ascertained by defuzzifying the relative closeness values.

In this paper endeavoured to discuss the significance of interval valued fuzzy sets and more importantly the magnitude of Linguistic Interval Valued Trapezoidal Fuzzy Number in section-2. Section-3 discusses the Arithmetic Behaviour on Linguistic Interval Valued Trapezoidal Fuzzy Numbers and the subsequent theorem. As far as the Section-4 is concerned, it deliberates the TOPSIS method using Interval Valued Trapezoidal Fuzzy Number. The subsequent Section-5 discourses the proposed critical path method putting into use Linguistic Interval Valued Trapezoidal fuzzy numbers followed by the numerical examples to substantiate the theory.

2. INTERVAL VALUED FUZZY SETS

In this segment we endeavoured to discourse interval valued Trapezoidal fuzzy number. In fuzzy sets theory, it is a lot difficult for an expert to precisely quantify their opinion as in interval [0, 1]. As a result, it is more appropriate to epitomize this degree of certainty by an interval. Wang and Li [20] demarcated valued fuzzy numbers and gave their extended operations. Built on the classification of interval valued fuzzy set [9] an interval valued on $(-\infty,\infty)$ is given by

$$A=\left\{\left(x,\left[\mu_A^L(x),\mu_A^U(x)\right]\right)\right\}\ \mu_A^L(x),\mu_A^U(x):X\to[0,1]\quad \forall x\in X,\mu_A^L\le\mu_A^U(x)$$

$$\tilde{\mu}_A(x)=\left[\mu_A^L(x),\mu_A^U(x)\right]\ A=\left\{\left(x,\tilde{\mu}_A(x)\right)\right\},x\in(-\infty,\infty)$$

Where $\mu_A^L(x)$ lower limit of the degree of the membership is function and $\mu_A^U(x)$ is the upper limit of the degree of the membership function.

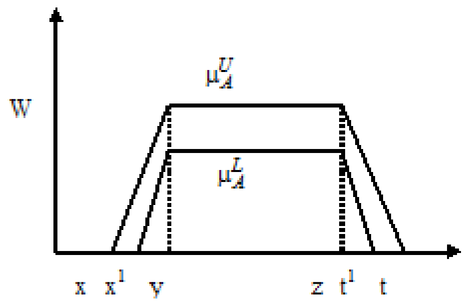

Fig.-1: Interval valued trapezoidal fuzzy number

Linguistic interval valued trapezoidal fuzzy number

A linguistic variable is a variable values of which are articulated in linguistic terms. The hypothesis of putting into use a linguistic variable comes very usable in dealing with state of affairs that are very complex or ill-defined to be reasonable styled in conventional quantitative. For instance, "weight" is a linguistic variable whose merits are very low, low, medium, high and very high as expounded in the Table-1 respectively. [9] Imparted an interval valued trapezoidal fuzzy number of the form $\left(\left(x, x'\right), y, z, \left(t', t\right); w\right)$ where $\left(x \le x' \le y \le z \le t' \le t \le 1 : 0 < w < 1\right)$.

3. ARITHMETIC BEHAVIOUR ON LINGUISTIC INTERVAL VALUED TRAPEZOIDAL FUZZY NUMBERS

Two linguistic interval valued trapezoidal fuzzy numbers are
$\tilde{A} = \left(\left(x_1, x_1'\right), y_1, z_1, \left(t_1', t_1\right); w_1\right)$ $\tilde{B} = \left(\left(x_2, x_2'\right), y_2, z_2, \left(t_2', t_2\right); w_2\right)$

The new arithmetic operation on linguistic interval valued trapezoidal fuzzy numbers as follows:

Addition of the linguistic interval valued trapezoidal fuzzy numbers:

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= \left(\left(x_1, x_1'\right), y_1, z_1, \left(t_1', t_1\right); w_1\right) \oplus \left(\left(x_2, x_2'\right), y_2, z_2, \left(t_2', t_2\right); w_2\right) \\ &= \left(\left(x_3, x_3'\right), y_3, z_3, \left(t_3', t_3\right); w_3\right) \end{aligned} \tag{1}$$

Where $w_3 = \min\left\{w_1, w_2\right\}$ and the

$$x_3 = x_1 + x_2 - x_1 x_2$$
$$x_3' = x_1' + x_2' - x_1' x_2'$$
$$y_3 = x_1' + x_2' + \left(y_1 - x_1'\right)\frac{w_3}{w_1} + \left(y_2 - x_2'\right)\frac{w_3}{w_2} - y_1 y_2$$
$$z_3 = t_1' + t_2' - \left(t_1' - z_1\right)\frac{w_3}{w_1} - \left(t_2' - z_2\right)\frac{w_3}{w_2} - z_1 z_2$$
$$t_3' = t_1' + t_2' - t_1' t_2'$$
$$t_3 = t_1 + t_2 - t_1 t_2$$

Multiplication of linguistic interval valued trapezoidal fuzzy numbers

$$\begin{aligned}\tilde{A} \otimes \tilde{B} &= \left(\left(x_1, x_1'\right), y_1, z_1, \left(t_1', t_1\right); w_1\right) \otimes \left(\left(x_2, x_2'\right), y_2, z_2, \left(t_2', t_2\right); w_2\right) \\ &= \left(\left(x_1 \times x_2, x_1' \times x_2'\right), y_1 \times y_2, z_1 \times z_2, \left(t_1' \times t_2', t_1 \times t_2\right); \min\left(w_1, w_2\right)\right) \end{aligned} \tag{2}$$

Division of linguistic interval valued trapezoidal fuzzy numbers

$$\begin{aligned}\tilde{A} / \tilde{B} &= \left(\left(x_1, x_1'\right), y_1, z_1, \left(t_1', t_1\right); w_1\right) / \left(\left(x_2, x_2'\right), y_2, z_2, \left(t_2', t_2\right); w_2\right) \\ &= \left(\left(x_4, x_4'\right), y_4, z_4, \left(t_4', t_4\right); \min\left(w_1, w_2\right)\right) \end{aligned} \tag{3}$$

Where $x_4 = \dfrac{x_1}{t_2}$, $x_4' = \dfrac{x_1'}{t_2'}$, $y_4 = \dfrac{y_1}{z_2}$, $z_4 = \dfrac{z_1}{y_2}$, $t_4' = \dfrac{t_1'}{x_2'}$ $t_4 = \dfrac{t_1}{x_2}$

Theorem: If $\tilde{A}$ and $\tilde{B}$ are two linguistic interval valued trapezoidal fuzzy numbers then $\tilde{A} \oplus \tilde{B}$, $\tilde{A} \otimes \tilde{B}$, $\tilde{A} / \tilde{B}$ are also linguistic interval valued trapezoidal fuzzy numbers.

Proof: Suppose that the two linguistic interval valued trapezoidal fuzzy numbers
$\tilde{A} = \left(\left(x_1, x_1'\right), y_1, z_1, \left(t_1', t_1\right); w_1\right)$ $\tilde{B} = \left(\left(x_2, x_2'\right), y_2, z_2, \left(t_2', t_2\right); w_2\right)$
where
$\left(x_1 \le x_1' \le y_1 \le z_1 \le t_1' \le t_1 \le 1 : 0 < w_1 < 1\right), \left(x_2 \le x_2' \le y_2 \le z_2 \le t_2' \le t_2 \le 1 : 0 < w_2 < 1\right)$

Now $\tilde{A} \oplus \tilde{B} = \left(\left(x_3, x_3'\right), y_3, z_3, \left(t_3', t_3\right); w_3\right)$

Where $x_3 = x_1 + x_2 - x_1 x_2 = x_1\left(1 - x_2\right) + x_2 \ge 0$

$x_3' = x_1' + x_2' - x_1' x_2' = x_1'\left(1 - x_2'\right) + x_2' \ge 0$

$$y_3 = x_1' + x_2' + (y_1 - x_1')\frac{w_3}{w_1} + (y_2 - x_2')\frac{w_3}{w_2} - y_1 y_2$$
$$= x_1'\left(1 - \frac{w_3}{w_1}\right) + x_2'\left(1 - \frac{w_3}{w_2}\right) + y_1\left(\frac{w_3}{w_1}\right) + y_2\left(\frac{w_3}{w_2}\right) - y_1 y_2 \geq 0$$
$$z_3 = t_1' + t_2' - (t_1' - z_1)\frac{w_3}{w_1} - (t_2' - z_2)\frac{w_3}{w_2} - z_1 z_2$$
$$= t_1'\left(1 - \frac{w_3}{w_1}\right) + t_2'\left(1 - \frac{w_3}{w_2}\right) + z_1\left(\frac{w_3}{w_1}\right) + z_2\left(\frac{w_3}{w_2}\right) - z_1 z_2 \geq 0$$

Also $1 - (t_1' + t_2' - t_1't_2') = (1 - t_1')(1 - t_2') \geq 0$
$1 - (t_1 + t_2 - t_1 t_2) = (1 - t_1)(1 - t_2) \geq 0$

So that $\tilde{A} \oplus \tilde{B}$ is linguistic interval valued trapezoidal fuzzy number.
It is clear that $\tilde{A} \otimes \tilde{B} == \left((x_1 \times x_2, x_1' \times x_2'), y_1 \times y_2, z_1 \times z_2, (t_1' \times t_2', t_1 \times t_2); \min(w_1, w_2)\right)$ is also a linguistic interval valued trapezoidal fuzzy number.

Consider the inequalities $0 \leq x \leq y \leq 1, 0 \leq a_1 \leq a_2 \leq 1$

We prove that $\frac{y}{a_1} \geq \frac{x}{a_2}$

Since $x \leq y$ and $0 \leq a_1 \leq 1$ implies that $a_1 x \leq a_1 y$, $0 \leq a_1 \leq a_2 \leq 1$ implies that $a_1 y \leq a_2 y$

So the relation becomes $a_1 x \leq a_1 y \leq a_2 y$ implies that $a_1 x \leq a_2 y$

Thus $\frac{y}{a_1} \geq \frac{x}{a_2}$ .With this consideration the following relation can be written as

$$\tilde{A} / \tilde{B} = \left((x_1, x_1'), y_1, z_1, (t_1', t_1); w_1\right) / \left((x_2, x_2'), y_2, z_2, (t_2', t_2); w_2\right)$$
$$= \left((x_4, x_4'), y_4, z_4, (t_4', t_4); \min(w_1, w_2)\right)$$

Where $x_4 = \frac{x_1}{t_2}$, $x_4' = \frac{x_1'}{t_2'}$, $y_4 = \frac{y_1}{z_2}$, $z_4 = \frac{z_1}{y_2}$, $t_4' = \frac{t_1'}{x_2'}$ $t_4 = \frac{t_1}{x_2}$

4. TOPSIS METHOD USING INTERVAL VALUED TRAPEZOIDAL FUZZY NUMBERS

Behzad Ashtiani *et al.* [8] extended TOPSIS method from Chen [5, 6] can be calculated as follows:

The decision group has K members, the k^{th} Decision makers rating and important weights are the i^{th} alternative on j^{th} criterion represented as

$\tilde{x}_{ij}^k = \left((a_{ij}^k, a_{ij}'^k), b_{ij}^k, c_{ij}^k, (d_{ij}'^k d_{ij}^k)\right)$ and $\tilde{w}_j^k = \left((w_{j1}^k, w_{j1}'^k), w_{j2}^k, w_{j3}^k, (w_{j4}'^k, w_{j4}^k)\right)$ respectively.

Where $i = 1, 2, ..., m$, $j = 1, 2, ..., n$ .The aggregated fuzzy rating $\tilde{x}_{ij}$ of alternatives (i) with respect to each criterion (j) are given by

$$\tilde{x}_{ij} = \left((a_{ij}, a_{ij}'), b_{ij}, c_{ij}, (d_{ij}', d_{ij})\right) \text{ Such that } a_{\bar{j}} = \min_k \left\{a_{ij}'^k\right\}$$
$$b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ij}^k,\ c_{ij} = \frac{1}{K}\sum_{k=1}^{K} c_{ij}^k,\ d_{ij} = \max_k \left\{d_{ij}^k\right\}.$$

The aggregated fuzzy weights $\tilde{w}_{ij}$ of each criterion are calculated as

$$\tilde{w}_j^k = \left(\left(w_{j1}, w'_{j1}\right), w_{j2}, w_{j3}, \left(w'_{j4}, w_{j4}\right)\right)$$

Where $w'_{j1} = \min_k \left\{w_{jk1}\right\}, w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2}$

$$w_{j3} = \frac{1}{K}\sum_{k=1}^{K} w_{jk3}, w_{j4} = \max_k \left\{w_{jk4}\right\}$$

Fuzzy multi criteria group decision making problems can be expressed in matrix form

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \dots & \dots & \tilde{x}_{ij} & \dots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{array}$$

$$\tilde{W} = \left(\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n\right)$$

Where $\tilde{x}_{ij}$ and $\tilde{w}_j$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ Are the linguist variables which can be represented by Interval valued trapezoidal fuzzy number as of the form $\tilde{x}_{ij} = \left(\left(a_{ij}, a'_{ij}\right), b_{ij}, c_{ij}, \left(d'_{ij}, d_{ij}\right)\right)$ and

$$\tilde{w}_j = \left(\left(w_{j1}, w'_{j1}\right), w_{j2}, w_{j3}, \left(w'_{j4}, w_{j4}\right)\right)$$

Step-1: The normalized performance rating can be calculated as

$$\tilde{r}_{ij} = \left(\left(\frac{a_{ij}}{d_j^+}, \frac{a'_{ij}}{d_j^+}\right), \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \left(\frac{d'_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+}\right)\right) \quad j \in B \tag{4}$$

$$\tilde{r}_{ij} = \left(\left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{d'_{ij}}\right), \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \left(\frac{a_j^-}{a'_{ij}}, \frac{a_j^-}{a_{ij}}\right)\right) \quad j \in C \tag{5}$$

Where B in Eq.4 and C in Eq.5 are the sets of benefit criteria and cost criteria, respectively.

$$d_j^+ = \max_i \left(d_{ij}\right)$$

$$a_j^- = \min_i \left(a_{ij}\right)$$

Step-2: By considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as

$\tilde{V} = \left[\tilde{v}_{ij}\right]_{m \times n}$ Where $\tilde{v}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j$

From the definition the product can be written as

$$\tilde{v}_{ij} = \left[\left(\tilde{r}_{1_{ij}} \times \tilde{w}_{1_j}, \tilde{r}'_{1_{ij}} \times \tilde{w}'_{1_j}\right), \tilde{r}_{2_{ij}} \times \tilde{w}_{2_j}, \tilde{r}_{3_{ij}} \times \tilde{w}_{3_j}, \left(\tilde{r}'_{4_{ij}} \times \tilde{w}'_{4_j}, \tilde{r}_{4_{ij}} \times \tilde{w}_{4_j}\right)\right]$$

$$= \left[\left(m_{ij}, m'_{ij}\right), n_{ij}, p_{ij}, \left(q_{ij}, q'_{ij}\right)\right] \tag{6}$$

Step-3: Positive Ideal and negative ideal solutions can be defined as

$$A^+ = \left[(1,1), 1, 1, (1,1)\right] \tag{7}$$

$$A^- = \left[(0,0), 0, 0, (0,0)\right] \tag{8}$$

Step-4: The distance of each alternative from the ideal alternative $\left[D_{i1}^{+}, D_{i2}^{+}\right]$ can be calculated as

$$D_{i1}^{+} = \sqrt{\frac{1}{6}\left[\left(m_{ij}-1\right)^2 + 2\left(n_{ij}-1\right)^2 + 2\left(p_{ij}-1\right)^2 + \left(q_{ij}-1\right)^2\right]} \tag{9}$$

$$D_{i2}^{+} = \sqrt{\frac{1}{6}\left[\left(m'_{ij}-1\right)^2 + 2\left(n_{ij}-1\right)^2 + 2\left(p_{ij}-1\right)^2 + \left(q'_{ij}-1\right)^2\right]} \tag{10}$$

The separation from the negative ideal solution $\left[D_{i1}^{-}, D_{i2}^{-}\right]$ can be calculated as

$$D_{i1}^{-} = \sqrt{\frac{1}{6}\left[\left(m_{ij}-0\right)^2 + 2\left(n_{ij}-0\right)^2 + 2\left(p_{ij}-0\right)^2 + \left(q_{ij}-0\right)^2\right]} \tag{11}$$

$$D_{i2}^{-} = \sqrt{\frac{1}{6}\left[\left(m'_{ij}-0\right)^2 + 2\left(n_{ij}-0\right)^2 + 2\left(p_{ij}-0\right)^2 + \left(q'_{ij}-0\right)^2\right]} \tag{12}$$

Step-5: The relative closeness can be calculated as follows:

$$RC_1 = \frac{D_{i2}^{-}}{D_{i2}^{+} + D_{i2}^{-}}, RC_2 = \frac{D_{i1}^{-}}{D_{i1}^{+} + D_{i1}^{-}} \tag{13}$$

The final value of RC_i^{*} determined as

$$RC_i^{*} = \frac{RC_1 + RC_2}{2} \tag{14}$$

5. PROPOSED CRITICAL PATH METHOD USING LINGUISTIC INTERVAL VALUED TRAPEZOIDAL FUZZY NUMBERS

In this subdivision a meticulous approaches to invent the critical path problem under fuzzy ambiance. In this paper, times are postulated as linguistic variables which are exemplified as positive Trapezoidal fuzzy numbers. Now a comprehensive elucidation is offered for the intended procedure. A gigantic project can be segregated into many activities. Regulate the extent and predilection relations of these activities. The preference affiliation of these endeavours may be visualized in the fuzzy project network. Thus, drawing the antecedence project network, which denote activities also ascertaining the yardsticks which are quintessential to pick out the critical path in the project network under these criteria. A path is one of the routes from initial node to the concluding node. Pinpoint all the paths in the fuzzy project network that start with a opening event and conclude with an ending event. Under the deliberation of each of the paths, one particular path is chosen as a critical path. Decide on the appropriate linguistic variables for qualitative criteria and trapezoidal fuzzy number for quantitative benchmark in order to acquire the fuzzy appraisal of activity under each criterion. Then, all linguistic assessments are transfigured into apt trapezoidal fuzzy numbers. The length of a path is the sum of extent of activities on the path, therefore add up trapezoidal fuzzy number to authenticate the final measurement value of each criterion for the paths. The extent of the longest path of the entire project network is the duration of the project. The longest path of the project network is also called the critical path. Hence, construct the fuzzy-decision matrix in which its substitute is the paths that start with the initial event and finishes with the concluding event. To reckon the completion time of the project, we necessitate concluding the critical path in the project network under different yardsticks. To single out the appropriate alternative, a critical path under different benchmarks, apply fuzzy TOPSIS procedure can deal with the ratings of both qualitative as well as quantitative criteria.

Table-1: Linguistic variables

Linguistic variables	Trapezoidal Fuzzy Number
Very low(V.L)	[(0.01,0.05),0.07,0.08,(0.1,0.15)]
Low(L)	[(0.02,0.05),0.1,0.15,(0.25,0.35)]
Medium low(M.L)	[(0.1,0.15),0.3,0.4,(0.45,0.55)]
Medium(M)	[(0.25,0.35),0.4,0.5,(0.65,0.75)]
Medium high(M.H)	[(0.45,0.55),0.6,0.7,(0.8,0.95)]
High(H)	[(0.55,0.75),0.8,0.9,(0.95,1)]
Very high(V.H)	[(0.85,0.95),1,1,(1,1)]

Numerical Example

The procedure put forward operates on some network. An activity is rifted into many a particle, with the intention that a conclusion could be scoped through the longest way in the shortest feasible while. Draw the significance of the network. It is paraded in Fig.2. Decision makers put to use the variable of linguistic weighing as depicted in the Table-1 to quantify the importance of the criteria. The decision maker has exhibited his assessment construed on linguistic variables for time is described in Table-2.

The table given here under represents as time rating of the activity by decision makers under various yardsticks.

Table-2: Rating of the activity by decision makers under Various criteria

Activity	Time
$1-2$	Very low
$1-3$	Medium
$1-4$	High
$2-5$	Medium Low
$3-5$	Medium
$3-6$	Low
$4-7$	Very High
$5-8$	Very low
$6-8$	Medium High
$7-8$	High

Next the transfiguration of linguistic Evaluation in to Trapezoidal Fuzzy Numbers is elaborated in the Table -3.

The following table embodies as transformed Linguistic Evaluation into linguistic interval valued Trapezoidal Fuzzy Numbers.

Table-3: Converted Linguistic Evaluation in to Trapezoidal Fuzzy Numbers.

Activity	Time
$1-2$	[(0.01,0.05),0.07,0.08,(0.1,0.15)]
$1-3$	[(0.25,0.35),0.4,0.5,(0.65,0.75)]
$1-4$	[(0.55,0.75),0.8,0.9,(0.95,1)]
$2-5$	[(0.1,0.15),0.3,0.4,(0.45,0.55)]
$3-5$	[(0.25,0.35),0.4,0.5,(0.65,0.75)]
$3-6$	[(0.02,0.05),0.1,0.15,(0.25,0.35)]
$4-7$	[(0.85,0.95),1,1,(1,1)]
$5-8$	[(0.01,0.05),0.07,0.08,(0.1,0.15)]
$6-8$	[(0.45,0.55),0.6,0.7,(0.8,0.95)]
$7-8$	[(0.55,0.75),0.8,0.9,(0.95,1)]

Add up interval valued trapezoidal fuzzy numbers in order to acquire the values of final assessment of each time benchmark for paths that get under way with the starting event and comes to a finale with the ending event. Successively create a fuzzy decision matrix as illustrated in Table-4.

The following table denotes Fuzzy –decision matrix.

Table-4: Fuzzy –decision matrix

path	**Time** W=[(0.7,0.75),0.9,0.9,(0.95,1))]
1-2-5-8	[(0.118,0.233),0.395,0.492,(0.555,0.675)]
1-3-5-8	[(0.443,0.599), 0.666,0.77,(0.890,0.947)]
1-3-6-8	[(0.596,0.722),0.784,0.873,(0.948,0.991)]
1-4-7-8	[(0.969,0.997),1,1,(1,1)]

The construction of fuzzy decision matrix is followed by the building up of the normalized fuzzy decision matrix by applying Eq-5. In order to obtain normalized fuzzy decision matrix the procedure is described in Table-5.
The following Table-5 represents Normalized Fuzzy –decision matrix.

Table-5: Normalized Fuzzy –decision matrix

Paths	**Time** W= [(0.7,0.75),0.9,0.9,(0.95,1))]
1-2-5-8	[(0.175,0.213),0.239,0.299,(0.506,1)]
1-3-5-8	[(0.125,0.133),0.153,0.177,(0.197,0.266)]
1-3-6-8	[(0.119,0.124)0.135,0.151,(0.163,0.198)]
1-4-7-8	[(0.118,0.118),0.118,0.118,(0.1183,0.122)]

The next step to follow is the construction of weighted normalized fuzzy decision matrix employing Eq.6. The weight of the criteria is supposed as
W= [(0.7, 0.75), 0.9, 0.9, (0.95, 1))].

The following Table-6 represents Weighted Normalized Fuzzy –decision matrix.

Table-6: Weighted Normalized Fuzzy –decision matrix

Paths	**Time**
1-2-5-8	[(0.1225,0.15975),0.2151,0.2691,(0.4807,1)]
1-3-5-8	[(0.0875,0.09975), 0.1377,0.1593,(0.18715,0.266)]
1-3-6-8	[(0.0833,0.093),0.1215,0.1359 ,(0.15485,0.198)]
1-4-7-8	[(0.0826,0.0885), 0.1062, 0.1062 (0.112385,0.122)]

Positive Ideal and negative ideal solutions defined in Eq-6. Eq-7 respectively

$$A^{+} = \left[(1,1),1,1,(1,1) \right] \quad A^{-} = \left[(0,0),0,0,(0,0) \right]$$

Using Eq-9,Eq-10, Eq-11,Eq-12, the distance of each alternative from Positive Ideal Alternative $\left[\mathrm{D}_{i1}^{+}, \mathrm{D}_{i2}^{+} \right]$ and also the distance of each alternative from negative Ideal Solution is $\left[\mathrm{D}_{i1}^{-}, \mathrm{D}_{i2}^{-} \right]$ expressed in Table-7.

The following Table-7 represents the distance from the ideal solution and negative ideal solution.

Table-7: Distance from the ideal solution and negative ideal solution

Paths	$\left[\mathrm{D}_{i1}^{+}, \mathrm{D}_{i2}^{+} \right]$	$\left[\mathrm{D}_{i1}^{-}, \mathrm{D}_{i2}^{-} \right]$
1-2-5-8	[0.72,0.74]	[0.29,0.46]
1-3-5-8	[0.84,0.85]	[0.15,0.17]
1-3-6-8	[0.86,0.87]	[0.13,0.14]
1-4-7-8	[0.8951,0.8957	[0.104,0.105]

The relative proximity coefficient can be gauged using the Eq.13 and Eq.14.

Ultimately evaluate the distance of each path from interval valued FPIS and FNIS in regard to each criterion as exhibited in the Table-8. Then workout and of the four conceivable paths and then measure the proximity of coefficient of each path as cited in the Table.8. in accordance with the closeness of four paths, know that the first path (1-2-5-8) is the critical path under the benchmark of time.

Table-8: Interval of relative closeness

paths	Interval of relative closeness $\left[RC_1, RC_2\right]$	$RC_i^* = \frac{RC_1 + RC_2}{2}$
1-2-5-8	[0.2797,0.3897]	**0.3347** ←
1-3-5-8	[0.1488,0.1651]	0.15695
1-3-6-8	[0.1283,0.1362]	0.13225
1-4-7-8	[0.10448,0.1054]	0.10494

The following Fig-2 represents Fuzzy Project network

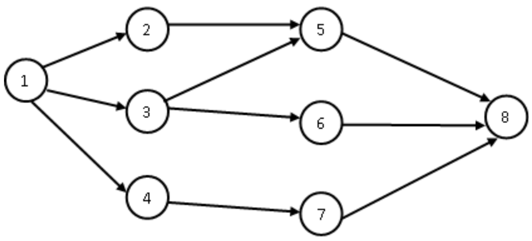

Fig.-2: Fuzzy Project network

7. CONCLUSION

Preceding studies on critical path investigation have established the use of fuzzy numbers as activity times in a fuzzy project network. In this paper, interval valued linguistic fuzzy sets are used to find fuzzy critical path in a fuzzy project network. The interval valued linguistic fuzzy variables can afford a more precise activity times in project network analysis. New arithmetic operations on linguistic interval valued fuzzy numbers are defined to get accurate fuzzy critical path in a fuzzy project network using Technique for order preference by similarity to ideal solution (TOPSIS) method.

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