

## SPECIAL ADDITION IN POLYNOMIALS USING NUMERICAL METHODS

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(Received On: 18-04-15; Revised & Accepted On: 31-05-15)

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### ABSTRACT

*It is the method of finding equation of a curve that approximates a given set of data on the basis of this mathematical equation, predictions can be made in many statistical investigation. Through this equation we can find the next output of any periodic function.*

$$Y_{q+2} = \frac{(n+1)}{1!} Y_{q+1} - \frac{n(n+1)}{2!} Y_q + \frac{n(n+1)(n-1)}{3!} Y_{q-1} - \frac{n(n+1)(n-1)(n-2)}{4!} Y_{q-2} + \dots$$

Where n = degree of the polynomial.

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### 1. INTRODUCTION

Let

X: x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub>, x<sub>4</sub>, x<sub>5</sub>,...

Y: y<sub>1</sub>, y<sub>2</sub>, y<sub>3</sub>, y<sub>4</sub>, y<sub>5</sub>,...

y<sub>1</sub>, y<sub>2</sub>, y<sub>3</sub>,...,y<sub>n</sub> be the corresponding x<sub>i</sub> values of degree two polynomial. So we can find y<sub>n</sub> value by without finding the polynomial. In this case x<sub>i</sub> should be equally spaced.

For a two degree polynomial

$$Y_{q+3} = 3Y_{q+2} - 3Y_{q+1} + Y_q$$

#### Example:

$$f(x) = x^2 + 4x + 2$$

X:-5, -4, -3, -2, -1, 0, 1, 2, 3, 4,...

Y: 7, 2, -1, -2, -1, 2, 7, 14, 23, 34...

$$(y_1)(y_2)(y_3)(y_4)(y_5)(y_6)(y_7)\dots$$

$$\text{Let } y_7 = 3y_6 - 3y_5 + y_4$$

$$7 = 3(2) - 3(-1) + 2$$

### 2. Proof: How can find

$$Y_{q+3} = 3Y_{q+2} - 3Y_{q+1} + Y_q$$

Let

X: X<sub>1</sub> X<sub>2</sub> X<sub>3</sub> X<sub>4</sub> X<sub>5</sub> X<sub>6</sub> ... X<sub>n</sub>

Y: y<sub>1</sub> y<sub>2</sub> y<sub>3</sub> y<sub>4</sub> y<sub>5</sub> y<sub>6</sub> ... y

$$\Delta y_3 = y_3 - y_2$$

$$\Delta y_4 = y_4 - y_3$$

$$\text{So } y_5 = \Delta^2 y_4 + \Delta y_4 + y_4$$

$$= \Delta y_4 - \Delta y_3 + y_2$$

$$= 3y_4 - 3y_3 + y_2$$

Proof of the above relation

$$Y_{q+3} = 3Y_{q+2} - 3Y_{q+1} + Y_q$$

$$\begin{aligned} a(x+3)^2 + b(x+3) + c &= 3[a(x+2)^2 + b(x+2) + c] - 3[a(x+1)^2 + b(x+1) + c] + [a(x)^2 + b(x) + c] \\ &= 3ax^2 + 12ax + 12a + 3bx + 6b + 3c - 3ax^2 - 6ax - 3a - 3bx - 3b - 3c + ax^2 + bx + c \\ &= (ax^2 + 6ax + 9a) + (bx + 3b) + c \\ &= a(x+3)^2 + b(x+3) + c \end{aligned}$$

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### Evaluation of other polynomial

- 1) Evaluation for 1 degree polynomials

$$y_3 = 2y_2 - y_1$$

- 2) Evaluation for 3 degree polynomials

$$y_5 = 4y_4 - 6y_3 + 4y_2 - y_1$$

- 3) Evaluation for 4 degree polynomials

$$Y_6 = 5y_5 - 10y_4 + 10y_3 - 5y_2 + y_1$$

- 4) Evaluation for 5 degree polynomials

$$Y_7 = 6y_6 - 15y_5 + 20y_4 - 15y_3 + 6y_2 - y_1$$

So from above, we can create a general equation for n degree polynomial

$$Y_{q+2} = \frac{(n+1)}{1!} y_{q+1} - \frac{n(n+1)}{2!} y_q + \frac{n(n+1)(n-1)}{3!} y_{q-1} - \frac{n(n+1)(n-1)(n-2)}{4!} y_{q-2} + \dots$$

Where n = degree of the polynomial

Introducing a new series

$$\frac{(n+1)}{1!} - \frac{n(n+1)}{2!} + \frac{n(n+1)(n-1)}{3!} - \frac{n(n+1)(n-1)(n-2)}{4!} + \dots = 1 \quad (1)$$

We adding and subtracting 1 in equation (1)

$$\begin{aligned} 1 &= \frac{(n+1)!}{(n+1)!} \\ &= 1 - \frac{(n+1)!}{(n+1)!} + \frac{(n+1)!}{1!(n!)!} - \frac{(n+1)!}{2!(n-1)!} + \frac{(n+1)!}{3!(n-2)!} - \frac{(n+1)!}{4!(n-3)!} + \dots \\ &= 1 - [n + 1]c_0 + n + 1c_1(-1) + n + 1c_2(-1)^2 + n + 1c_3(-1)^3 + \dots \\ &= 1 - (1 - 1)^{n+1} = 1 \end{aligned}$$

$$\text{i.e. } \sum_{q=1}^n \frac{(-1)^{q+1}(n+1)!}{q![n-(q-1)]!} = 1$$

### 3. THE PROOF OF GENERAL EQUATION

$$(x + (y+2))^n = (n+1)(x+(y+1))^n - \frac{n(n+1)}{2!} (x+y)^n + \frac{n(n+1)(n-1)}{3!} (x+(y-1))^n + \dots$$

By binomial theorem

$$(x + (y+2))^n = x^n - nx^{n-1} (y+2) + \frac{n(n-1)}{2!} nx^{n-2} (y+2)^2 + \dots$$

I.e.;

$$\begin{aligned} (x + (y+2))^n &= \frac{(n+1)}{1!} [x^n + nx^{n-1} (y+1) + \frac{n(n-1)}{2!} nx^{n-2} (y+1)^2 + \dots] - \frac{n(n+1)}{2!} [x^n + nx^{n-1} (y) + \frac{n(n-1)}{2!} nx^{n-2} (y)^2 + \dots] \\ &\quad + \frac{n(n+1)(n-1)}{3!} [x^n + nx^{n-1} (y-1) + \frac{n(n-1)}{2!} nx^{n-2} (y-1)^2 + \dots] \dots \\ &= x^n \left[ \frac{(n+1)}{1!} - \frac{n(n+1)}{2!} + \frac{n(n+1)(n-1)}{3!} - \frac{n(n+1)(n-1)(n-2)}{4!} + \dots \right] \\ &\quad - x^{n-1} (y+2) \left[ n \left[ \frac{(n+1)}{1!} - \frac{n(n+1)}{2!} + \frac{n(n+1)(n-1)}{3!} - \frac{n(n+1)(n-1)(n-2)}{4!} + \dots \right] \right] \\ &\quad - x^{n-2} (y+2) \left[ \frac{n(n-1)}{2!} \left[ \frac{(n+1)}{1!} - \frac{n(n+1)}{2!} + \frac{n(n+1)(n-1)}{3!} - \frac{n(n+1)(n-1)(n-2)}{4!} + \dots \right] \right] + \dots \end{aligned}$$

We know

$$\left[ \frac{(n+1)}{1!} - \frac{n(n+1)}{2!} + \frac{n(n+1)(n-1)}{3!} - \frac{n(n+1)(n-1)(n-2)}{4!} + \dots \right] = 1$$

$$(x + (y+2))^n = x^n - nx^{n-1} (y+2) + \frac{n(n-1)}{2!} nx^{n-2} (y+2)^2 + \dots$$

**Source of support: Nil, Conflict of interest: None Declared**

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