In this paper we introduce $g^{**}$-closed sets in bitopological spaces. Properties of these sets are investigated and we introduce three new bitopological spaces namely, $(i, j)$-$**T_{1/2}$ spaces, $(i, j)$-$T_{1/2}^{**}$ space and $(i, j)$-$*T_{1/2}^{*}$ spaces.

Key words: $(i, j)$-$g^{**}$-closed sets, $(i, j)$-$**T_{1/2}$ spaces, $(i, j)$-$T_{1/2}^{**}$ spaces and $(i, j)$-$*T_{1/2}^{*}$ spaces.

1. INTRODUCTION

A triple $(X, \tau_1, \tau_2)$ where $X$ is a non-empty set and $\tau_1$ and $\tau_2$ are topologies in $X$ is called a bitopological space and Kelly [5] initiated the study of such spaces. In 1985, Fukutake [2] introduced the concepts of $g$-closed sets in bitopological spaces. M.K.R.S. Veerakumar [11] introduced and studied the concepts of $g^*$-closed sets and $g^*$-continuity in topological spaces. Sheik John. M and Sundaram. P [8] introduced and studied the concepts of $g^*$-closed sets in bitopological spaces in 2002. The purpose of this paper is to introduce the concepts of $g^{**}$-closed sets, $(i, j)$-$**T_{1/2}$ spaces, $(i, j)$-$T_{1/2}^{**}$ spaces and $(i, j)$-$*T_{1/2}^{*}$ spaces in bitopological spaces and investigate some of their properties.

2. PRELIMINARIES

Definition 2.1 A subset $A$ of a topological space $(X, \tau)$ is said to be
1. a pre-open set [7] if $A \subseteq \text{int}(\text{cl}(A))$ and a preclosed set if $\text{cl}(\text{int}(A)) \subseteq A$.
2. a semi-open set [6] if $A \subseteq \text{cl}(\text{int}(A))$ and and semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.
3. a regular open set [9] if $A = \text{int}(\text{cl}(A))$.
4. a generalized closed set [7] (briefly g-closed set) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
5. a generalized star closed set [11] (briefly $g^*$-closed set) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is g-open in $(X, \tau)$.
6. a generalized star star closed set [10] (briefly $g^{**}$-closed set) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $(X, \tau)$.

If $A$ is a subset of $X$ with topology $\tau$, then the closure of $A$ is denoted by $\tau - \text{cl}(A)$ or $\text{cl}(A)$, the interior of $A$ is denoted by $\tau - \text{int}(A)$ or $\text{int}(A)$ and the complement of $A$ in $X$ is denoted by $A^c$.

For a subset of $(X, \tau, \tau_i, \tau_j)$, $\tau_i$-cl($A$) (resp. $\tau_i$-$\text{int}(A)$) denote the closure (resp. interior)of $A$ with respect to the topology $\tau_i$. We denote the family of all g-open(resp.$g^*$-open) subsets of $X$ with respect to the topology $\tau_i$ by $GO(X, \tau_i)$ (resp. $G^*O(X, \tau_i)$ and the family of all $\tau_j$-closed sets is denoted by the symbol $F_j$ we mean the pair of topologies $(\tau^j, \tau^j)$.

Definition 2.2 A subset $A$ of a topology $(X, \tau, \tau_j)$ is called
1. $(i, j)$-g-closed[2] if $\tau_j$-$\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $\tau_j$.
2. $(i, j)$-rg-closed[1] if $\tau_j$-$\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $\tau_j$.
3. $(i, j)$-gpr-closed[4] if $\tau_j$-$\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $\tau_j$.
4. $(i, j)$-wg-closed[3] if $\tau_j$-$\text{cl}(\tau_i$-$\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $\tau_i$.

Definition 2.3 A bitopological space $(X, \tau_i, \tau_j)$ is called
1. an $(i, j)$-$T_{1/2}$ space[2] if every $(i, j)$-g-closed set is $\tau_j$-closed.
2. a strongly pairwise $T_{1/2}$ space[2] if it is both $(1, 2)$ $T_{1/2}$ and $(2, 1)$ $T_{1/2}$.
3. an $(i, j)$-$T_{1/2}^{**}$ space[8] if every $(i, j)$-g*-closed set is $\tau_j$-closed.
4. a strongly pairwise $T_{1/2}^{**}$ space[8] if it is both $(1, 2)$ $T_{1/2}^{**}$ and $(2, 1)$ $T_{1/2}^{**}$.
5. an $(i, j)$-$*T_{1/2}^{*}$ space[8] if every $(i, j)$-g-closed set is $g^*$-closed.
6. a strongly pairwise $*T_{1/2}^{*}$ space[8] if it is both $(1, 2)$ $*T_{1/2}^{*}$ and $(2, 1)$ $*T_{1/2}^{*}$.
3. (i, j) – g**-closed sets

In this section we introduce the concept of (i, j)-g**-closed sets in bitopological spaces.

Definition 3.1: A subset A of a topological space \( (X, \tau_1, \tau_2) \) is said to be an (i, j) - g**-closed set if \( \tau_j - cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in G*O(X, \tau_j) \). We denote the family of all (i, j)-g**-closed sets in \( (X, \tau_1, \tau_2) \) by \( D**(i, j). \)

Remark 3.2: By setting \( \tau_1 = \tau_2 \) in definition (3.1), a (i, j) - g**-closed set is a g**-closed set.

Proposition 3.3: Every \( \tau_j - \text{closed} \) subset of \( (X, \tau_1, \tau_2) \) is (i, j) - g**-closed.

The converse of the above propositions is not true as seen in the following example.

Example 3.4: Let \( X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, c\}, X\} \) and \( \tau_2 = \{\phi, \{a\}, X\} \). Then the set \( A = \{b\} \) is (1, 2) - g** - closed but not \( \tau_2 - \text{closed} \) in \( (X, \tau_1, \tau_2) \).

Proposition 3.5: If \( A \) is both \( \tau_i - \text{open} \) and (i, j) - g**-closed then \( A \) is \( \tau_j - \text{closed} \).

Proposition 3.6: In a Bitopological space \( (X, \tau_1, \tau_2) \) every (i, j) - g**-closed set is

(i) \( g - \text{closed} \)
(ii) \( \tau_i - \text{closed} \)
(iii) \( gpr - \text{closed} \)
(iv) \( wg - \text{closed} \).

The following examples show that the converse of the above proposition are not true.

Example 3.7: Let \( X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, b\}, X\} \) and \( \tau_2 = \{\phi, \{a\}, X\} \). Then the set \( A = \{c\} \) is (1, 2) - g**-closed but not (1, 2) - \( g** - closed \).

Example 3.8: Let \( X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, b\}, X\} \) and \( \tau_2 = \{\phi, \{a\}, X\} \). Then the subset \( A = \{c\} \) is (1, 2) - gpr - closed but not (1, 2) - g**-closed.

Proposition 3.9: Every (i, j) - g**-closed set is (i, j) - g**-closed.

The converse of the above need not be true.

Example 3.10: Let \( X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\} \) and \( \tau_2 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\} \). Then the subset \( A = \{b\} \) is (1, 2) - g**-closed but not (1, 2) - g**-closed.

Proposition 3.11: If \( A, B \in D**(i, j) \), then \( A \cup B \in D**(i, j) \).

Remark 3.12: The intersection two (i, j) - g**-closed set need not be (i, j) - g**-closed as seen from the following example.

Example 3.13: Let \( X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{b, c\}, X\} \) and \( \tau_2 = \{\phi, \{b\}, \{a, c\}, \{b, c\}, \{a, c\}, X\} \). Let \( A = \{a, b\} \) and \( B = \{b, c\} \). Then \( A \) and \( B \) are (2,1)- g** - closed sets but \( A \cap B = \{b\} \) is not a (2,1) - g**-closed set.

Remark 3.14: \( D**(1,2) \) is generally not equal to \( D**(2,1) \).

Example 3.15: In Example (3.13), \( A = \{b\} \notin D**(2,1) \) but \( A \in D**(1,2) \).
Proposition 3.16: If $\tau_1 \subseteq \tau_2$, in $(X, \tau_1, \tau_2)$ then $D^{**}(2,1) \subseteq D^{**}(1,2)$.

The converse of the above need not be true as seen in the following example.

Example 3.17: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{b\}, \{a\}, \{c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{b\}, X\}$ where $\tau_1 \not\subseteq \tau_2$ but $D^{**}(2,1) \subseteq D^{**}(1,2)$.

Proposition 3.18: For each element $x$ of $(X, \tau_1, \tau_2)$, $\{x\}$ is either $g^*-$closed or $X - \{x\}$ is $(i, j)$ - $g^{**}-closed$.

Proposition 3.19: If $A$ is $(i, j)$ - $g^{**}-closed$, then $\tau_j - cl(A) - A$ contains no non-empty $\tau_i - g^*-$closed set.

Proof: Let $A$ be $(i, j)$ - $g^{**}-closed$ and let $F$ be a $\tau_i - g^*-$closed set such that $F \subseteq \tau_j - cl(A) - A$. Since $A \in D^{**}(i, j)$, we have $\tau_j - cl(A) \subseteq F^C$.

Therefore $F \subseteq (\tau_j - cl(A)) \cap (\tau_j - cl(A))^C = \emptyset$. Therefore $F = \emptyset$.

The converse of the above two propositions need not be true as it is seen in the following example.

Example 3.20: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{b, c\}, X\}$.

Let $A = \{b\}$. Then $\tau_2 - cl(A) \setminus A = \{c\}$ is not $\tau_i - g^*-$closed. i.e. $\tau_2 - cl(A) \setminus A$ contains no non-empty $\tau_i - g^*-$closed set but $A = \{b\}$ is not $(1, 2)$ - $g^{**}-closed$.

Theorem 3.22: If A is $(i, j)$ - $g^{**}-closed$ in $(X, \tau_i, \tau_j)$ then A is $\tau_j - closed$ if and only if $\tau_j - cl(A) \setminus A$ is $\tau_i - g^*-$closed.

Proof: Necessity: If A is $\tau_j - closed$ then $\tau_j - cl(A) = A$ that is $\tau_j - cl(A) \setminus A = \emptyset$ and hence it is $\tau_i - g^*-$closed $g^*-$closed.

Sufficiency: If $\tau_j - cl(A) \setminus A$ is $\tau_i - g^*-$closed then by proposition (3.19), $\tau_j - cl(A) \setminus A = \emptyset$. Therefore A is $\tau_j - g^*-$closed.

Theorem 3.23: If A is an $(i, j)$ - $g^{**}-closed$ set of $(X, \tau_i, \tau_j)$ such that $A \subseteq B \subseteq \tau_j - cl(A)$ then B is also an $(i, j)$ - $g^{**}-closed$ set of $(X, \tau_i, \tau_j)$.

Proof: Let $B \subseteq U$ and U be $\tau_i - g^*-$open. Then $A \subseteq U$ and $\tau_j - cl(A) \subseteq U$. since A is $(i, j) - g^{**}-closed$, $B \subseteq \tau_j - cl(A)$ implies $\tau_j - cl(B) \subseteq \tau_j - cl(A)$ and hence $\tau_j - cl(B) \subseteq U$.

Therefore B is $(i, j) - g^{**}-closed$.

Proposition 3.24: In a Bitopological space $(X, \tau_i, \tau_j)$, $G^*O(X, \tau_i) \subseteq F_j$ if and only if every subset of X is an $(i, j) - g^{**}-closed$ set.

Proof: Suppose $G^*O(X, \tau_i) \subseteq F_j$. Let A be a subset of X such that $A \subseteq U$ where $U \in G^*O(X, \tau_i)$. Then $\tau_j - cl(A) \subseteq \tau_j - cl(U) = U$ and hence A is $(i, j) - g^{**}-closed$. Conversely, suppose that every subset of X
is \((i, j)\) - \(g^{**} - closed\). Let \(U \in G \bullet O(X, \tau_i)\). Since \(U\) is \((i, j)\) - \(g^{**} - closed\), we have \(\tau_j - cl(U) \subseteq U\). Therefore \(U = \tau_j - cl(U)\) and hence \(U \in F_j\).

Therefore \(G \bullet O(X, \tau_i) \subseteq F_j\).

The following figure illustrates the relationships with the other closed sets:

4. \((i, j)\) - \(T_{1/2}^{**}\) - spaces \((i, j)\) - \(T_{1/2}^{**}\) - \(spaces\) \((i, j)\) - \({}^*T_{1/2}^{**}\) - \(spaces\)

In this section we introduce three new bitopological spaces \((i, j)\) - \(T_{1/2}^{**}\) - \(spaces\), \((i, j)\) - \(T_{1/2}^{**}\) - \(spaces\) and \((i, j)\) - \({}^*T_{1/2}^{**}\) - \(spaces\).

**Definition 4.1**: A bitopological space \((X, \tau_1, \tau_2)\) is said to be an \((i, j)\) - \(T_{1/2}^{**}\) - \(space\) if every \((i, j)\) - set is \((i, j)\) - \(g^{**} - closed\) \(g^{*}\) - \(closed\).

**Proposition 4.2**: Every \((i, j)\) - \(T_{1/2}^{**}\) - \(space\) is a \((i, j)\) - \(T_{1/2}^{**}\) - \(space\) but not conversely.

**Example 4.3**: Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, \{a\}, \{a, c\}, X\}\) and \(\tau_2 = \{\phi, \{a\}, \{b, c\}, X\}\). Then \((X, \tau_1, \tau_2)\) is a \((1, 2)\) - \(T_{1/2}^{**}\) - \(space\) but not a \((1, 2)\) - \(T_{1/2}^{**}\) - \(space\) since \(A = \{b\}\) is \((1, 2)\) - \(g\) - \(closed\) but not \(\tau_2\) - \(closed\).

**Remark 4.4**: \(A \text{ (1, 2)} \text{ - } T_{1/2}^{**}\) - \(space\) need not be a \((1, 2)\) - \(T_{1/2}^{**}\) - \(space\) true as it is seen in the following example.

**Example 4.5**: Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, \{a\}, X\}\) and \(\tau_2 = \{\phi, \{a\}, \{b, c\}, X\}\). Then \((X, \tau_1, \tau_2)\) is a \((1, 2)\) - \(T_{1/2}^{**}\) - \(space\) but not a \((1, 2)\) - \(T_{1/2}^{**}\) - \(space\) since \(A = \{b\}\) is \((1, 2)\) - \(g^{**}\) - \(closed\) but not \((1, 2)\) - \(g^{*}\) - \(closed\).

**Definition 4.6**: A bitopological space \((X, \tau_1, \tau_2)\) is said to be a \((i, j)\) - \(T_{1/2}^{**}\) - \(space\) if every \((i, j)\) - \(g^{**}\) - \(closed\) \(g^{*}\) - \(closed\).

**Proposition 4.7**: If \((X, \tau_1, \tau_2)\) is a \((i, j)\) - \(T_{1/2}^{**}\) - \(space\) then it is a \((i, j)\) - \(T_{1/2}^{**}\) - \(space\).

The converse of the above is not be true as seen in the following example.

**Example 4.8**: In example (4.5), \((X, \tau_1, \tau_2)\) is \((1, 2)\) - \(T_{1/2}^{**}\) - \(space\) but not \((1, 2)\) - \(T_{1/2}^{**}\) - \(space\). Since \(A = \{b\}\) is \((1, 2)\) - \(g^{**}\) - \(closed\) but not \(\tau_2\) - \(closed\).
Proposition 4.9: If a bitopological space \((X, \tau_1, \tau_2)\) is a \((1, 2) - T_{1/2}^{**} - space\) then it is both \((1, 2) - T_{1/2}^{**} - space\) and \((1, 2) - T_{1/2}^{**} - space\).

Proof follows from propositions (4.2) and (4.7).

Proposition 4.10: Every \((i, j) - T_{1/2}^{**} - space\) is \((i, j) - T_{1/2}^{**} - space\) but not conversely.

Example 4.11: Let \(X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, b\}, X\} \) and \(\tau_2 = \{\phi, \{a, b\}, X\}\). Then \((X, \tau_1, \tau_2)\) is a \((i, j) - T_{1/2}^{**} - space\) but not a \((i, j) - T_{1/2}^{**} - space\) since \(A = \{a, b\}\) is \((1, 2) - g^{**} - closed\) but not \(\tau_2 - closed\).

Definition 4.12: A bitopological space \((X, \tau_1, \tau_2)\) is said to be a strongly pairwise \(T_{1/2}^{**} - space\) if it is both \((1, 2) - T_{1/2}^{**} - space\) and \((2, 1) - T_{1/2}^{**} - space\).

Definition 4.13: A bitopological space \((X, \tau_1, \tau_2)\) is said to be a strongly pairwise \(T_{1/2}^{**} - space\) if it is both \((1, 2) - T_{1/2}^{**} - space\) and \((2, 1) - T_{1/2}^{**} - space\).

Proposition 4.14: If \((X, \tau_1, \tau_2)\) is a strongly pairwise \(T_{1/2}^{**} - space\) then it is a strongly pairwise \(T_{1/2}^{**} - space\) but not conversely.

Example 4.15: Let \(X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, b\}, X\} \) and \(\tau_2 = \{\phi, \{b\}, \{a, c\}, \{b, c\}, X\}\).

Then \((X, \tau_1, \tau_2)\) is a strongly pairwise \(T_{1/2}^{**} - space\) but not a strongly pairwise \(T_{1/2}^{**} - space\) since \(A = \{c\}\) is \((1, 2) - g - closed\) but not \(\tau_2 - closed\).

Proposition 4.16: If \((X, \tau_1, \tau_2)\) is a strongly pairwise \(T_{1/2}^{**} - space\) then it is a strongly pairwise \(T_{1/2}^{**} - space\) but not conversely.

Example 4.17: Let \(X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, b\}, X\} \) and \(\tau_2 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}\).

Then \((X, \tau_1, \tau_2)\) is a strongly pairwise \(T_{1/2}^{**} - space\) but not a strongly pairwise \(T_{1/2}^{**} - space\) since \(A = \{a, c\}\) is \((2, 1) - g^{**} - closed\) but not \(\tau_1 - closed\). Therefore \((X, \tau_1, \tau_2)\) is not a \((2, 1) - T_{1/2}^{**} - space\) and hence it is not a strongly pairwise \(T_{1/2}^{**} - space\).

Proposition 4.18: The following conditions are equivalent in a bitopological space \((X, \tau_1, \tau_2)\)

(i) \((X, \tau_1, \tau_2)\) is a \((i, j) - T_{1/2}^{**} - space\).

(ii) Every singleton of \(X\) is either \(\tau_i - g^{*} - closed\) or \(\tau_j - open\).

Proof: (i) \(\rightarrow\) (ii). Let \((X, \tau_1, \tau_2)\) be an \((i, j) - T_{1/2}^{**} - space\). Let \(x \in X\) and suppose \(\{x\}\) is not \(\tau_i - g^{*} - closed\). Then \(X - \{x\}\) is not \(\tau_i - g^{*} - open\). Therefore \(X - \{x\}\) is a \((i, j) - g^{**} - closed\) set of \((X, \tau_1, \tau_2)\) since \((X, \tau_1, \tau_2)\) is a \((i, j) - T_{1/2}^{**} - space\). \(X - \{x\}\) is \(\tau_j - closed\). Therefore \(\{x\}\) is \(\tau_j - open\).

(ii) \(\rightarrow\) (i). Let \(A\) be a \((i, j) - g^{**} - closed\) set of \((X, \tau_1, \tau_2)\). \(A \subseteq \tau_j - cl(A)\). Let \(x \in \tau_j - cl(A)\). By (ii), \(\{x\}\) is either \(\tau_i - g^{*} - closed\) or \(\tau_j - open\).
Case (i): Let \( \{x\} \) be \( \tau_j - g^* \text{closed} \). Suppose \( x \notin A \), then \( \tau_j - cl(A) - A \) contains a non-empty \( \tau_j - g^* \text{closed} \) set \( \{x\} \), which is a contradiction to propositions (3.22). Therefore \( x \in A \).

Case (ii): Suppose \( \{x\} \) is \( \tau_j - \text{open} \). Since \( x \in \tau_j - \text{cl}(A) \), \( \{x\} \cap A \neq \emptyset \). Therefore we have \( x \in A \). This in both cases, we conclude that \( A \) is \( \tau_j - \text{closed} \). Hence \( (X, \tau_1, \tau_2) \) is an \( (i, j) - T_{1/2}^* - \text{closed} \).

Definition 4.19: A space \( (X, \tau_1, \tau_2) \) is called a \( (i, j) - T_{1/2}^* - \text{space} \) if every \( (i, j) - g - \text{closed} \) set of \( (X, \tau_1, \tau_2) \) is \( (i, j) - g^* - \text{closed} \).

Definition 4.20: A bitopological space \( (X, \tau_1, \tau_2) \) is said to be a strongly pairwise \( T_{1/2}^* - \text{space} \) if it is both \( (1, 2) - T_{1/2}^* - \text{spaces} \) and \( (2, 1) - T_{1/2}^* - \text{spaces} \).

Proposition 4.21: Every \( (i, j) - T_{1/2}^* - \text{space} \) is an \( (i, j) - T_{1/2}^* - \text{space} \) but not conversely.

Example 4.22: In example (4.3), \( (X, \tau_1, \tau_2) \) is \( (1, 2) - T_{1/2}^* - \text{space} \) but not \( (2, 1) - T_{1/2}^* - \text{space} \) since \( A = \{b\} \) is \( (1, 2) - g - \text{closed} \) but not \( \tau_2 - \text{closed} \).

Remark 4.23: \( (i, j) - T_{1/2}^* - \text{space} \) and \( (i, j) - T_{1/2}^* - \text{spaces} \) are independent as seen in the following example.

Example 4.24: In example (4.5), \( (X, \tau_1, \tau_2) \) is \( (i, j) - T_{1/2}^* - \text{spaces} \) but not \( (i, j) - T_{1/2}^* - \text{space} \) since \( A = \{b\} \) is \( (1, 2) - g^* - \text{closed} \) but not \( \tau_2 - \text{closed} \).

Example 4.25: Let \( X = \{a, b, c\} \), \( \tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \) and \( \tau_2 = \{\phi, \{a\}, \{a, b\}, X\} \). Then \( (X, \tau_1, \tau_2) \) is \( (1, 2) - T_{1/2}^* - \text{space} \) but not \( T_{1/2}^* - \text{space} \) since \( A = \{a, c\} \) is \( (1, 2) - g^* - \text{closed} \) but not \( \tau_2 - \text{closed} \).

Proposition 4.26: A space \( (X, \tau_1, \tau_2) \) is a \( (i, j) - T_{1/2}^* - \text{space} \) if and only if it is both \( (i, j) - T_{1/2}^* - \text{space} \) and \( (i, j) - T_{1/2}^* - \text{space} \).

Proof: Let \( A \) be an \( (i, j) - g - \text{closed} \) set in \( X \). Since \( (X, \tau_1, \tau_2) \) is a \( (i, j) - T_{1/2}^* - \text{spaces} \), \( A \) is \( (i, j) - g^* - \text{closed} \) set in \( X \). Again since \( (X, \tau_1, \tau_2) \) is \( (i, j) - T_{1/2}^* - \text{space} \), \( A \) is \( (i, j) - g^* - \text{closed} \).

Therefore every \( (i, j) - g - \text{closed} \) set of \( (X, \tau_1, \tau_2) \) is \( (i, j) - g^* - \text{closed} \). Hence \( (X, \tau_1, \tau_2) \) is an \( (i, j) - T_{1/2}^* - \text{space} \).

Conversely suppose \( (X, \tau_1, \tau_2) \) be an \( (i, j) - T_{1/2}^* - \text{space} \). Let \( A \) be an \( (i, j) - g^* - \text{closed} \) set of \( (X, \tau_1, \tau_2) \). Then from (i) of proposition (3.6), \( A \) is \( (i, j) - g - \text{closed} \). Since \( (X, \tau_1, \tau_2) \) is \( T_{1/2}^* - \text{space} \), \( A \) is \( (i, j) - g^* - \text{closed} \) and hence \( (X, \tau_1, \tau_2) \) is \( (i, j) - T_{1/2}^* - \text{space} \). Let \( A \) be an \( (i, j) - g - \text{closed} \) set. Since \( (X, \tau_1, \tau_2) \) is \( T_{1/2}^* - \text{space} \), \( A \) is \( (i, j) - g^* - \text{closed} \). Then by

Proposition (3.9), \( A \) is \( (i, j) - g^* - \text{closed} \). Therefore \( (X, \tau_1, \tau_2) \) is \( (i, j) - T_{1/2}^* - \text{space} \).

Proposition 4.27: A space \( (X, \tau_1, \tau_2) \) is strongly pairwise \( T_{1/2}^* - \text{space} \) if and only if it is both strongly pairwise \( T_{1/2}^* - \text{space} \) and strongly pairwise \( T_{1/2}^* - \text{space} \). Proof follows from proposition 4.26.
Proposition 4.28: Every strongly pairwise \( T_{1/2} \) – space is strongly pairwise \( T_{1/2}^* \) – spaces but not conversely. Proof follows from proposition 4.21.

Example 4.29: In example (4.22), \((X, \tau_1, \tau_2)\) is strongly pairwise \( T_{1/2}^* \) – space but not a strongly pairwise \( T_{1/2} \) – space since \( A = \{b\} \) is (1, 2) - \( g \) – closed but not \( \tau_2 \) – closed.

The results in this section can be represented in the following figure:

[Diagram: A \( \rightarrow \) B represents A implies B but not conversely and A \( \leftrightarrow \) B represents A and B are independent.]

REFERENCES

[10] Pauline Mary Helen, M. Veronica Vijayan, Ponnuthai Selvarani, \( g^* \)-closed sets in topological spaces (accepted).

Source of support: Nil, Conflict of interest: None Declared