ON $\psi\alpha g$-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of set called $\psi\alpha g$-closed sets in topological spaces and also we introduce a new type of functions called $\psi\alpha g$-continuous functions and $\psi\alpha g$-irresolute functions. Further we introduce a $\psi\alpha g$-homeomorphism and studied the group structure of $\psi\alpha g$-homeomorphism in topological spaces.

Keywords: $\psi\alpha g$-closed set, $\psi\alpha g$-continuous function, $\psi\alpha g$-irresolute function and $\psi\alpha g$-homeomorphism.

1. INTRODUCTION

O. Njastad [11] introduces the concept of $\alpha$-closed sets in topological spaces. The notion of $\psi$-closed sets is introduced by MKRS Veerakumar [17]. In this chapter, we introduce a new class of notion, namely, $\psi\alpha g$-closed sets for topological spaces. As applications of $\psi\alpha g$-closed sets, we introduce and study some new spaces, namely $\psi\alpha gT_{1/2}$ space, $\psi\alpha gT_{s}$ space and $\psi\alpha gT_{\psi}$ spaces. Further we introduce and study $\psi\alpha g$-continuous and $\psi\alpha g$- irresolute maps. For a topological Space $(X, \tau)$, we define groups $\psi\alpha g-h(X, \tau)$, $\psi\alpha g-ch(X, \tau)$ and that contain the group $h(X, \tau)$ whose elements are all homeomorphisms from $(X, \tau)$ into itself.

2. PRELIMINARIES

In this section we recall some of the basic definitions.

Definition 2.1: A subset $A$ of space $(X, \tau)$ is called

(i) semi open set [9] if $A \subseteq \text{cl(int}(A))$.
(ii) semi $\text{Pre}$ open set [1] if $A \subseteq \text{cl(int(cl}(A)))$.
(iii) $\text{pre}$ open set [10] if $A \subseteq \text{int(cl}(A))$.
(iv) $\alpha$-open set [11] if $A \subseteq \text{int(cl(int}(A)))$.
(v) regular open set [12] if $A = \text{int(cl}(A))$.

Definition 2.2: A subset $A$ of space $(X, \tau)$ is called

(i) generalized closed (briefly $g$-closed) set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open set in $(X, \tau)$
(ii) generalized semi-closed (briefly $g$-$\text{Pre}$-closed) set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open set in $(X, \tau)$
(iii) $\alpha$-generalized closed (briefly $\alpha g$-closed) set if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open set in $(X, \tau)$
(iv) generalized pre-closed (briefly $gp$-closed) set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open set in $(X, \tau)$
(v) generalized semi-pre-closed (briefly $gsp$-closed) set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open set in $(X, \tau)$
(vi) generalized pre-regular-closed (briefly $gpr$-closed) set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open set in $(X, \tau)$
(vii) $g^{\#}$-closed set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha g$-open set in $(X, \tau)$
(viii) $g^{\#}$-$\text{Pre}$-closed set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha g$-open set in $(X, \tau)$
(ix) $\alpha \psi$-closed set if $\alpha \psi \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open set in $(X, \tau)$

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Definition 2.3: A space $(X, \tau)$ is said to be
(i) $\alpha$-space if every $\alpha$-closed set is closed.
(ii) $T_\alpha$-space if every $gsp$-closed set is $gsp$-closed.
(iii) pre-regular $T_{1/2}$-space if every $gsp$-closed set is $gsp$-closed.
(iv) $T_\#$-space if every $g\#s$-closed set is closed.
(v) semi-$T_1/3$-space if every $\psi$-closed set is semi-closed.
(vi) $\alpha\psi T_{1/2}$-space if every $\psi\psi\psi\psi$-closed set is closed.

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called
(i) semi-continuous if $f^{-1}(V)$ is a semi-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(ii) $\alpha$-continuous if $f^{-1}(V)$ is a $\alpha$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(iii) $gp$-continuous if $f^{-1}(V)$ is a $gp$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(iv) $gp^r$-continuous if $f^{-1}(V)$ is a $gp^r$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(v) $gsp$-continuous if $f^{-1}(V)$ is a $gsp$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(vi) $g\#$-continuous if $f^{-1}(V)$ is a $g\#$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(vii) $g\#s$-continuous if $f^{-1}(V)$ is a $g\#s$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(viii) $\psi\psi\psi\psi$-continuous if $f^{-1}(V)$ is a $\psi\psi\psi\psi$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

3. $\psi\alpha\psi$-CLOSED SETS IN TOPOLOGICAL SPACES

We introduce the following definition

Definition 3.1: A subset $A$ of a space $(X, \tau)$ is $\psi\alpha\psi$-closed set if $\psi\cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $ag$-open set in $(X, \tau)$.

Theorem 3.2: Every closed set is $\psi\alpha\psi$-closed set.

Proof: Let $A$ be an closed set in $(X, \tau)$, $U$ be an $ag$-open set containing $A$. Since $A$ is closed, we have $cl(A) = A$, $\psi\cl(A) \subseteq cl(A) = A \subseteq U$ and hence $A$ is $\psi\alpha\psi$-closed set.

The converse of the above theorem is need not be true as it can be seen in the following example.

Example 3.3: Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{b\}$ is a $\psi\alpha\psi$-closed set but not an closed set.

Theorem 3.4: Every $\alpha$-closed set and $g\#$-closed is a $\psi\alpha\psi$-closed set.

Proof: Obvious.

The converse of the above theorem are need not be true as it can be seen in the following example.

Example 3.5: Let $X=Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{b\}, \{b, c\}\}$. A set $\{b\}$ is $\psi\alpha\psi$-closed set but not an $\alpha$-closed and $g\#$-closed set.

Theorem 3.6: Every semi-closed and $g\#s$-closed set is $\psi\alpha\psi$-closed set.

Proof: Obvious.

The following example shows that the converse of the above theorem are need not be true in general.

Example 3.7: Let $X=Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{b\}, \{b, c\}\}$. A set $\{b, c\}$ is $\psi\alpha\psi$-closed set but not an semi-closed and $g\#s$-closed set.

Theorem 3.8: Every $\psi\alpha\psi$-closed set is $gp$-closed, $gsp$-closed and $gpr$-closed set.

Proof: The proof is obvious. So the class of $\psi\alpha\psi$-closed set is properly contained in the class of $gp$-closed (resp. $gsp$-closed, $gpr$-closed) set.

The converse of the above theorem are need not be true as it can be seen in the following example.

Example 3.9: Let $X=Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{a,b\}\}$. A set $\{a, c\}$ is an $gp$-closed, $gsp$-closed and $gpr$-closed set but not an $\psi\alpha\psi$-closed set.
Theorem 4.10: Every $\psi_{ag}$-closed set is $\alpha g$-closed set.

Proof: The converse of the above theorem is need not true in general as it can be seen from the following example.

Example 3.11: Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. A set $\{a, b\}$ is an $\alpha g$-closed set but not an $\psi_{ag}$-closed set.

4. APPLICATIONS OF $\psi_{ag}$-CLOSED SETS

As applications of $\psi_{ag} T_{1/2}$-closed sets, two new spaces $\psi_{ag} T_{c}$-spaces and $\psi_{ag} T_{e}$-spaces are introduced.

We introduce the following definition.

Definition 4.1: A space $(X, \tau)$ is called $\psi_{ag} T_{1/2}$ space if every $\psi_{ag}$-closed set is closed.

Theorem 4.2: Every $\psi_{ag} T_{1/2}$ space is a $T_{#b}$-space and $\alpha$-space but not conversely.

Proof: The space of the following examples are $T_{#b}$-space and $\alpha$-space but not a $\psi_{ag} T_{1/2}$ space.

Example 4.3: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. $C(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\psi_{ag} - C(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is a $T_{#b}$-space but not a $\psi_{ag} T_{1/2}$ space.

Example 4.4: Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. $C(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}\}$, $G^1 SC(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\psi_{ag} - C(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is an $\alpha$-space but not a $\psi_{ag} T_{1/2}$ space.

Theorem 4.5: Every pre regular $T_{1/2}$ space is a $\psi_{ag} T_{1/2}$ space but not conversely.

Proof: A $\psi_{ag} T_{1/2}$ space is need not be $T_{1/2}$ in general as it can be seen from the following example.

Example 4.6: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$. $C(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}\}$, $G^1 SC(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\psi_{ag} - C(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is a $\psi_{ag} T_{1/2}$ space but not a pre regular $T_{1/2}$ space.

Theorem 4.7: If $(X, \tau)$ is a $\psi_{ag} T_{1/2}$ space, then for each $x \in X$, $\{x\}$ is either $\alpha$-closed or open.

Proof: Suppose that $(X, \tau)$ is a $\psi_{ag} T_{1/2}$ space. Let $x \in X$ and assume that $\{x\}$ is not a $\alpha$-closed set. Then $X \setminus \{x\}$ is not a $\alpha$-open set. This implies that $X \setminus \{x\}$ is a $\psi_{ag}$-closed set, since $X$ is the only $\alpha$-open set containing $X \setminus \{x\}$. Since $(X, \tau)$ is a $\psi_{ag} T_{1/2}$ space. $X \setminus \{x\}$ is a closed set or equivalently $\{x\}$ is open.

The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.8: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$. The set $\{a\}$ is an open set of $(X, \tau)$. $\{b\}$ and $\{c\}$ are $\alpha$-closed subsets of $(X, \tau)$. But $(X, \tau)$ is not a $\psi_{ag} T_{1/2}$ space.

Since $\psi_{ag} - c(X, \tau) = \{X, \emptyset, \{b\}, \{c\}\}$ and $C(X, \tau) = \{X, \emptyset, \{b\}, \{b, c\}\}$.

Definition 4.9: A space $(X, \tau)$ is called $\psi_{ag} T_{c}$-space if every $\psi_{ag}$-closed set is $\alpha$-closed.

Theorem 4.10: Every $\psi_{ag} T_{1/2}$ space is a $\psi_{ag} T_{c}$-space but not conversely.

Proof: The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.11: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$. $C(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{b, c\}\}$, $\alpha C(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{b, c\}\}$ and $\psi_{ag} - C(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is a $\psi_{ag} T_{c}$-space but not a $\psi_{ag} T_{1/2}$ space.

Theorem 4.12: Every $\psi_{ag} T_{a}$ space is a $\alpha$-space and $T_{#b}$-space but not conversely.

Proof: A $\alpha$-space and $T_{#b}$-space is need not be $\psi_{ag} T_{a}$ in general as it can be seen from the following example.

Example 4.13: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. $C(X, \tau) = \{X, \emptyset, \{a\}\}$, $\alpha C(X, \tau) = \{X, \emptyset, \{a\}\}$ and $\psi_{ag} - C(X, \tau) = \{X, \emptyset, \{a\}\}$. Thus the space $(X, \tau)$ is a $\alpha$-space but not a $\psi_{ag} T_{a}$ space.

Example 4.14: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. $C(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}\}$, $\alpha C(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\psi_{ag} - C(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is a $T_{#b}$-space but not a $\psi_{ag} T_{a}$ space.
Example 4.16: Let $X=\{a, b, c\}$ and $\tau=\{X, \phi, \{a\}, \{a, b\}, \{b, c\}\}$. The set $\{b\}$ is an open set of $(X, \tau)$. $\{a\}$ and $\{c\}$ are $\alpha$-closed subsets of $(X, \tau)$. But $(X, \tau)$ is not a $\psi_{ag}T_{1/2}$ space. Since $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $\alpha C(X, \tau)=\{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. Thus the space $(X, \tau)$ is a $\psi_{ag}T_{1/2}$-space but not a $\psi_{ag}T_{1}$-space.

Theorem 4.15: If $(X, \tau)$ is a $\psi_{ag}T_{a}$ space, then for each $x \in X$, $\{x\}$ is either $\alpha$-closed or $\alpha$-open.

Proof: Suppose that $(X, \tau)$ is a $\psi_{ag}T_{a}$ space. Let $x \in X$ and assume that $\{x\}$ is not a $\alpha$-closed set. Then $X-\{x\}$ is not a $\alpha$-open set. This implies that $X-\{x\}$ is a $\psi_{ag}$ -closed set, since $X$ is the only $\alpha$-open set containing $X-\{x\}$. Since $(X, \tau)$ is a $\psi_{ag}T_{a}$ space, $X-\{x\}$ is a $\alpha$-closed set or equivalently $\{x\}$ is $\alpha$-open.

The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.17: A space $(X, \tau)$ is called $\psi_{ag}T_{a}$ space if every $\psi_{ag}$-closed set is semi closed.

Theorem 4.18: Every $\psi_{ag}T_{1/2}$ space is a $\psi_{ag}T_{a}$ space but not conversely.

Proof: The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.19: Let $X=\{a, b, c\}$ and $\tau=\{X, \phi, \{a\}, \{a, b\}\}$. $C(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\}$, $SC(X, \tau)=\{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\}$ and $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is a $\psi_{ag}T_{a}$-space but not a $\psi_{ag}T_{1/2}$-space.

Theorem 4.20: Every $\psi_{ag}T_{a}$ space is a $\psi_{ag}T_{1/2}$-space and $T_{\#}$-space but not conversely.

Proof: Obvious.

The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.21: Let $X=\{a, b, c, d\}$ and $\tau=\{X, \phi, \{a\}, \{a, b\}\}$. $C(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}, \{c, d\}\}$, $SC(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}, \{c, d\}\}$ and $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}, \{c, d\}\}$. Thus the space $(X, \tau)$ is a $\psi_{ag}T_{a}$ space but not a $\psi_{ag}T_{1/2}$ space.

Example 4.22: Let $X=\{a, b, c\}$ and $\tau=\{X, \phi, \{a\}, \{b\}, \{c\}\}$. $C(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}\}$, $SC(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}\}$ and $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{b\}, \{c\}\}$. Thus the space $(X, \tau)$ is a $T_{\#}$-space but not a $\psi_{ag}T_{a}$ space.

Theorem 4.23: $\psi_{ag}T_{a}$ ness is independent of $aT_{d}$ ness.

Proof: This can be proved by the following examples.

Example 4.24: Let $X=\{a, b, c\}$ and $\tau=\{X, \phi, \{a\}, \{b, c\}\}$. $GC(X, \tau)=\{X, \phi, \{a\}, \{b, c\}\}$, $SC(X, \tau)=\{X, \phi, \{a\}, \{b, c\}\}$, $\alpha GC(X, \tau)=P(X)$, and $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is a $aT_{a}$-space but not a $\psi_{ag}T_{a}$ space.

Example 4.25: Let $X=\{a, b, c\}$ and $\tau=\{X, \phi, \{a\}, \{b, c\}\}$. $GC(X, \tau)=\{X, \phi, \{a\}, \{b, c\}\}$, $SC(X, \tau)=\{X, \phi, \{a\}, \{b, c\}\}$, $\alpha GC(X, \tau)=\{X, \phi, \{a\}, \{b, c\}\}$ and $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{b, c\}\}$. Thus the space $(X, \tau)$ is a $\psi_{ag}T_{a}$-space but not a $aT_{a}$ space.

Theorem 4.26: If $(X, \tau)$ is a $\psi_{ag}T_{a}$ space, then for each $x \in X$, $\{x\}$ is either $\alpha$-closed or semi-open.

Proof: Suppose that $(X, \tau)$ is a $\psi_{ag}T_{a}$ space. Let $x \in X$ and assume that $\{x\}$ is not a $\alpha$-closed set. Then $X-\{x\}$ is not a $\alpha$-open set. This implies that $X-\{x\}$ is a $\psi_{ag}$ -closed set, since $X$ is the only $\alpha$-open set containing $X-\{x\}$. Since $(X, \tau)$ is a $\psi_{ag}T_{a}$ space, $X-\{x\}$ is a $\alpha$-closed set or equivalently $\{x\}$ is semi-open.

The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.27: Let $X=\{a, b, c\}$ and $\tau=\{X, \phi, \{a\}, \{b, c\}\}$. The set $\{b\}$ is an open set of $(X, \tau)$. $\{a\}$ and $\{c\}$ are $\alpha$-closed subsets of $(X, \tau)$. But $(X, \tau)$ is not a $\psi_{ag}T_{a}$ space. Since $\psi_{ag}-C(X, \tau)=\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $SC(X, \tau)=\{X, \phi, \{a\}, \{c\}, \{a, c\}\}$.
5. $\psi\alpha g$-CONTINUOUS MAPS AND $\psi\alpha g$-IRRRESOLUTE MAPS

Definition 5.1: A function $f: (X, \tau) \to (Y, \sigma)$ is called $\psi\alpha g$-continuous if $f^{-1}(V)$ is a $\psi\alpha g$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

Theorem 5.2: Every $\psi\alpha g$-continuous map is gp-continuous and gsp-continuous.

Proof: Let $V$ be closed set in $(Y, \sigma)$. Since $f$ is $\psi\alpha g$-continuous, $f^{-1}(V)$ is $\psi\alpha g$-closed set in $(X, \tau)$, we know that every $\psi\alpha g$-closed set is gp-closed (resp. gsp-closed), $f^{-1}(V)$ is gp-closed (resp. gsp-closed) set in $(X, \tau)$. Therefore $f$ is gp-continuous (resp. gsp-closed).

The converse of the above theorem are need not be true as it can be seen in the following example.

Example 5.3: Let $X=Y=[a,b,c]$ with $\tau=\{\emptyset, X, [a], [a,c]\}$ and $\sigma=\{\emptyset, \phi, X, [a]\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here $f$ is gp-continuous and gsp-continuous but not $\psi\alpha g$-continuous, since $f^{-1}([a,b]) = \{a, b\}$ is not in $\psi\alpha g$-closed set in $(X, \tau)$.

Theorem 5.4: Every $\psi\alpha g$-continuous map is gp-continuous and $\alpha\psi$-continuous.

Proof: By the Theorem 3.8 and Theorem 3.10, The converse of the above theorem are need not be true as it can be seen in the following example.

Example 5.5: Let $X=Y=[a, b, c]$ with $\tau=\{\emptyset, X, [a], [b,c]\}$ and $\sigma=\{\emptyset, \phi, X, [a]\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here $f$ is gp-continuous and $\alpha\psi$-continuous but not $\psi\alpha g$-continuous, since $f^{-1}(\{b\}) = \{b\}$ is not in $\psi\alpha g$-closed set in $(X, \tau)$.

Theorem 5.6: Every semi-continuous and $g^s$-continuous map is $\psi\alpha g$-continuous.

Proof: By the Theorem 3.6, The converse of the above theorem are need not be true as it can be seen in the following example.

Example 5.7: Let $X=Y=[a, b, c]$ with $\tau=\{\emptyset, X, [b], [b,c]\}$ and $\sigma=\{\emptyset, \phi, X, [a]\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here $f$ is $\psi\alpha g$-continuous but not semi-continuous and $g^s$-continuous, since $f^{-1}([b,c]) = \{b, c\}$ is not in semi closed and $g^s$-closed set in $(X, \tau)$.

Theorem 5.8: Every $\alpha$-continuous map is $\psi\alpha g$-continuous.

Proof: By the Theorem 3.4, The converse of the above theorem are need not be true as shown in the following example.

Example 5.9: Let $X=Y=[a, b, c]$ with $\tau=\{\emptyset, X, [a], [b]\}$ and $\sigma=\{\emptyset, \phi, X, [c]\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here $f$ is $\psi\alpha g$-continuous but not $\alpha$-continuous, since $f^{-1}([a,b]) = \{a, b\}$ is not in $\alpha$-closed set in $(X, \tau)$.

Theorem 5.10: Every $g^s$-continuous map is $\psi\alpha g$-continuous.

Proof: By the Theorem 3.4, The converse of the above theorem are need not be true as shown in the following example.

Example 5.11: Let $X=Y=[a, b, c]$ with $\tau=\{\emptyset, X, [a], [b], [a,b]\}$ and $\sigma=\{\emptyset, \phi, X, [a,c]\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here $f$ is $\psi\alpha g$-continuous but not $g^s$-continuous, since $f^{-1}([b]) = \{b\}$ is not in $g^s$-closed set in $(X, \tau)$.

Definition 5.12: A function $f: (X, \tau) \to (Y, \sigma)$ is called $\psi\alpha g$-irresolute if $f^{-1}(V)$ is a $\psi\alpha g$-closed set of $(X, \tau)$ for every $\psi\alpha g$-closed set $V$ of $(Y, \sigma)$.

Theorem 5.13: Every $\psi\alpha g$-irresolute map is $\psi\alpha g$-continuous.

Proof: Let $V$ be a closed set of $(Y, \sigma)$ and hence it is $\psi\alpha g$-closed set. Since $f$ is $\psi\alpha g$-irresolute, $f^{-1}(V)$ is a $\psi\alpha g$-closed set of $(X, \tau)$. Hence $f$ is a $\psi\alpha g$-continuous map.

The converse of the above theorem is need not be true by the following example.

Example 5.14: Let $X=Y=[a,b,c]$ with $\tau=\{X, X, [a,b], [a,b,c]\}$ and $\sigma=\{X, X, [a], [a,b], [a,b]\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Then $f$ is not $\psi\alpha g$-irresolute, since $[a]$ is a $\psi\alpha g$-closed set in $(Y, \sigma)$, but $f^{-1}([a]) = \{a\}$ is not a $\psi\alpha g$-closed set of $(X, \tau)$. However $f$ is $\psi\alpha g$-continuous.
**Corollary 7.4:** If $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ are $\psi \sigma g$-irresolute, then the composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is $\psi \sigma a g$-irresolute.

**Proof:** Let $V$ be a $\psi \sigma g$-closed set in $(Z, \eta)$. Since $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a $\psi \sigma g$-irresolute function, $g^{-1}(V)$ is a $\psi \sigma g$-closed set in $(Y, \sigma)$. Since $f$ is an $\psi \sigma g$-irresolute functions, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is an $\psi \sigma g$-closed set in $(X, \tau)$. Hence $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is an $\psi \sigma g$-irresolute functions.

**Example 5.16:** Let $X=\{a,b,c\}$ with $\tau=\{X,\phi,\{a\},\{a,c\}\}$ and $\sigma=\{X,\phi,\{a\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Then $f$ is not $\psi \sigma g$-irresolute, since $\{a, c\}$ is a $\psi \sigma g$-closed set in $(Y, \sigma)$, but $f^{-1}(\{a\}) = \{a, c\}$ is not a $\psi \sigma g$-closed set of $(X, \tau)$. However $f$ is $\psi \sigma a g$-continuous.

**Theorem 5.17:** Every $\psi \sigma g$-irresolute function is gp-continuous.

**Proof:** It follows from the fact that the Theorem 3.8. The following example supports that the converse of the above theorem is not true.

**Example 5.18:** Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\phi,\{a\}\}$ and $\sigma=\{X,\phi,\{a\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Then $f$ is not $\psi \sigma g$-irresolute, since $\{a, c\}$ is a $\psi \sigma g$-closed set in $(Y, \sigma)$, but $f^{-1}(\{a\}) = \{a, c\}$ is not a gp-closed set of $(X, \tau)$. Thus $f$ is not $\psi \sigma g$-irresolute, however $f$ is gp-continuous.

7. **$\psi \sigma$-HOMEOMORPHISM AND THEIR GROUP STRUCTURE**

**Definition 7.1:** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

(i) $\psi \sigma$-open is the image $f(U)$ is $\psi \sigma$-open in $(Y, \sigma)$ for every open set $U$ of $(X, \tau)$.

(ii) $\psi \sigma$-closed when the image $f(U)$ is $\psi \sigma$-closed in $(Y, \sigma)$ for every open set $U$ of $(X, \tau)$.

(iii) $\psi \sigma$-homeomorphism if $f$ is bijective and $f$ and $f^{-1}$ are $\psi \sigma g$-continuous.

(iv) $\psi \sigma$-homeomorphism if $f$ is bijective and $f$ and $f^{-1}$ are $\psi \sigma g$-continuous.

**Theorem 7.2:**

(i) Suppose that $f$ is a bijection, then the following conditions are equivalent.

(a) $f$ is a $\psi \sigma$-homeomorphism.

(b) $f$ is a $\psi \sigma g$-open and $\psi \sigma g$-continuous.

(c) $f$ is a $\psi \sigma$-closed and $\psi \sigma g$-continuous.

(ii) If $f$ is a homeomorphism, then $f$ and $f^{-1}$ are $\psi \sigma g$-irresolute.

(iii) Every $\psi \sigma$-homeomorphism is a $\psi \sigma g$-homeomorphism.

**Proof:**

(i) It is obvious.

(ii) First we prove that $f^{-1}$ is $\psi \sigma g$-irresolute. Let $A$ be a $\psi \sigma g$-closed set in $(X, \tau)$. To show $(f^{-1})^{-1} = f(A)$ is $\psi \sigma g$-closed in $(Y, \sigma)$. Let $U$ be a $\psi \sigma g$-open set such that $f(A) \subseteq U$.

Then $A = f^{-1}(f(A)) \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is $\psi \sigma g$-open. Since $A$ is $\psi \sigma g$-closed, $\psi \sigma g(A) \subseteq f^{-1}(U)$, we have $\psi \sigma g(f(A)) = f(\psi \sigma g(A)) \subseteq f(f^{-1}(U)) \subseteq U$ and so $f(A)$ is $\psi \sigma g$-closed. Thus $f^{-1}$ is $\psi \sigma g$-irresolute. Since $f^{-1}$ is also a homeomorphism $(f^{-1})^{-1} = f$ is $\psi \sigma g$-irre solute.

(iii) It is proved by theorem 5.13.

**Definition 7.3:** For a topological space $(X, \tau)$ we define the following three collections of functions.

(i) $\psi \sigma$-ch$(X, \tau) = \{f: f: (X, \tau) \rightarrow (X, \tau)\}$ is a $\psi \sigma g$-c-homeomorphism.

(ii) $\psi \sigma$-h$(X, \tau) = \{f: f: (X, \tau) \rightarrow (X, \tau)\}$ is a $\psi \sigma g$-homeomorphism.

(iii) $h(X, \tau) = \{f: f: (X, \tau) \rightarrow (X, \tau)\}$ is a homeomorphism.

**Corollary 7.4:** For a topological space $(X, \tau)$ the following properties hold.

(i) $h(X, \tau) \subseteq \psi \sigma g \circ \text{ch}(X, \tau) \subseteq \psi \sigma g \circ h(X, \tau)$.

(ii) The set $\psi \sigma g \circ \text{ch}(X, \tau)$ forms a group under composition of functions.

(iii) The group $h(X, \tau)$ is a subgroup of $\psi \sigma g \circ \text{ch}(X, \tau)$.

(iv) If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a $\psi \sigma g$-c-homeomorphism then it induces an isomorphism $f^*: \psi \sigma g \circ \text{ch}(Y, \sigma) \rightarrow \psi \sigma g \circ \text{ch}(X, \tau)$.

**Proof:**

(i) It is proved by using theorem 5.9 and theorem 5.13 and a fact that every continuous map is $\alpha$-continuous.

(ii) It is proved by using $g \circ f$ is $\psi \sigma g$-irresolute if both $f$ and $g$ are $\psi \sigma g$-irresolute, for any element $a, b \in \psi \sigma g \circ \text{ch}(X, \tau)$ the following binary operation $\omega: \psi \sigma g \circ \text{ch}(X, \tau) \times \psi \sigma g \circ \text{ch}(X, \tau) \rightarrow \psi \sigma g \circ \text{ch}(X, \tau)$ is well defined $\omega(a, b) = b \circ a$. 
(iii) By (i), \( h(X, \tau) \subseteq \psi ag - ch(X, \tau) \) and \( h(X, \tau) \neq \phi \). For any elements \( a, b \in h(X, \tau) \) and the binary operation \( \omega \) in (ii) it is shown that \( \omega(a, b^{-1}) = b^{-1} \circ a \in h(X, \tau) \).

(iv) We define \( f^*: \psi ag - ch(Y, \sigma) \rightarrow \psi ag - ch(Y, \sigma) \) by \( f^*(h) = f \circ h \circ f^{-1} \).

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