International Journal of Mathematical Archive-7(12), 2016, 99-102
IJMA Available online through www.ijma.info ISSN 2229 – 5046

# BIPOLAR VALUED FUZZY SUBSEMIRINGS OF A SEMIRING USING HOMOMORPHISM AND ANTI-HOMOMORPHISM

**V. SHANMUGAPRIYA*[1], K. ARJUNAN[2]**

[1]Department of Mathematics,
H. H. The Rajah's College, Pudukkottai – 622001, Tamil Nadu, India.

[2]Department of Mathematics,
H. H. The Rajah's College, Pudukkottai – 622001, Tamil Nadu, India.

*(Received On: 19-11-16; Revised & Accepted On: 19-12-16)*

***ABSTRACT***

*In this paper, we made an attempt to study the algebraic nature of bipolar valued fuzzy subsemirings under homomorphism and anti-homomorphism and prove some results on these.*

***Key Words:*** *Bipolar valued fuzzy set, bipolar valued fuzzy subsemiring, bipolar valued fuzzy normal subsemiring.*

## INTRODUCTION

In 1965, Zadeh [14] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [7]. Lee [9] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [9, 10]. Anitha.M.S., Muruganantha Prasad & K.Arjunan[1, 2] defined as Bipolar valued fuzzy subgroups of a group and homomorphism, antihomomorphism are used. We introduce the concept of bipolar valued fuzzy subsemiring under homomorphism, antihomomorphism and established some results.

## 1. PRELIMINARIES

**1.1 Definition:** A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = {< x, $A^+(x)$, $A^-(x)$ >/ x∈X}, where $A^+ : X \to [0, 1]$ and $A^- : X \to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

**1.2 Example:** A = {< a, 0.5, –0.3 >, < b, 0.1, –0.7 >, < c, 0.5, –0.4 >} is a bipolar valued fuzzy subset of X= {a, b, c}.

*Corresponding Author: V. Shanmugapriya*[1], [1]Department of Mathematics,*
*H. H. The Rajah's College, Pudukkottai – 622001, Tamil Nadu, India.*

**1.3 Definition:** Let R be a semiring. A bipolar valued fuzzy subset A of R is said to be a bipolar valued fuzzy subsemiring of R if the following conditions are satisfied,
(i) $A^+(x+y) \geq \min\{A^+(x), A^+(y)\}$
(ii) $A^+(xy) \geq \min\{A^+(x), A^+(y)\}$
(iii) $A^-(x+y) \leq \max\{A^-(x), A^-(y)\}$
(iv) $A^-(xy) \leq \max\{A^-(x), A^-(y)\}$ for all x and y in R.

**1.4 Example:** Let R = $Z_3$ = {0, 1, 2} be a semiring with respect to the addition modulo and multiplication modulo. Then A = {< 0, 0.5, –0.6 >, < 1, 0.4, –0.5 >, < 2, 0.4, –0.5 >} is a bipolar valued fuzzy subsemiring of R.

**1.5 Definition:** Let R be a semiring. A bipolar valued fuzzy subsemiring A of R is said to be a bipolar valued fuzzy normal subsemiring of R if $A^+(x+y) = A^+(y+x)$, $A^+(xy) = A^+(yx)$, $A^-(x+y) = A^-(y+x)$ and $A^-(xy) = A^-(yx)$ for all x and y in R.

**1.6 Definition:** Let R and $R^l$ be any two semirings. Then the function f: R → $R^l$ is said to be an antihomomorphism if f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x) for all x and y in R.

**1.7 Definition:** Let X and $X^l$ be any two sets. Let f : X→ $X^l$ be any function and let A be a bipolar valued fuzzy subset in X, V be a bipolar valued fuzzy subset in f(X) = $X^l$, defined by $V^+(y) = \sup_{x \in f^{-1}(y)} A^+(x)$ and $V^-(y) = \inf_{x \in f^{-1}(y)} A^-(x)$, for all x in X and y in $X^l$. A is called a preimage of V under f and is defined as $A^+(x) = V^+(f(x))$, $A^-(x) = V^-(f(x))$ for all x in X and is denoted by $f^{-1}(V)$.

## 2. SOME PROPERTIES

**2.1 Theorem:** Let R and $R^l$ be any two semirings. The homomorphic image of a bipolar valued fuzzy subsemiring of R is a bipolar valued fuzzy subsemiring of $R^l$.

**Proof:** Let f : R→ $R^l$ be a homomorphism. Let V = f(A) where A is a bipolar valued fuzzy subsemiring of R. We have to prove that V is a bipolar valued fuzzy subsemiring of $R^l$. Now for f(x), f(y) in $R^l$, $V^+(f(x)+f(y)) = V^+(f(x+y)) \geq A^+(x+y) \geq \min\{A^+(x), A^+(y)\} = \min\{V^+(f(x)), V^+(f(y))\}$ which implies that $V^+(f(x)+f(y)) \geq \min\{V^+(f(x)), V^+(f(y))\}$. And $V^+(f(x)f(y)) = V^+(f(xy)) \geq A^+(xy) \geq \min\{A^+(x), A^+(y)\} = \min\{V^+(f(x)), V^+(f(y))\}$ which implies that $V^+(f(x)f(y)) \geq \min\{V^+(f(x)), V^+(f(y))\}$. Also $V^-(f(x)+f(y)) = V^-(f(x+y)) \leq A^-(x+y) \leq \max\{A^-(x), A^-(y)\} = \max\{V^-(f(x)), V^-(f(y))\}$ which implies that $V^-(f(x)+f(y)) \leq \max\{V^-(f(x)), V^-(f(y))\}$. And $V^-(f(x)f(y)) = V^-(f(xy)) \leq A^-(xy) \leq \max\{A^-(x), A^-(y)\} = \max\{V^-(f(x)), V^-(f(y))\}$ which implies that $V^-(f(x)f(y)) \leq \max\{V^-(f(x)), V^-(f(y))\}$. Hence V is a bipolar valued fuzzy subsemiring of $R^l$.

**2.2 Theorem:** Let R and $R^l$ be any two semirings. The homomorphic preimage of a bipolar valued fuzzy subsemiring of $R^l$ is a bipolar valued fuzzy subsemiring of R.

**Proof:** Let f: R → $R^l$ be a homomorphism. Let V = f(A) where V is a bipolar valued fuzzy subsemiring of $R^l$. We have to prove that A is a bipolar valued fuzzy subsemiring of R. Let x and y in R. Now $A^+(x+y) = V^+(f(x+y)) = V^+(f(x)+f(y)) \geq \min\{V^+(f(x)), V^+(f(y))\} = \min\{A^+(x), A^+(y)\}$ which implies that $A^+(x+y) \geq \min\{A^+(x), A^+(y)\}$. And $A^+(xy) = V^+(f(xy)) = V^+(f(x)f(y)) \geq \min\{V^+(f(x)), V^+(f(y))\} = \min\{A^+(x), A^+(y)\}$ which implies that $A^+(xy) \geq \min\{A^+(x), A^+(y)\}$. Also $A^-(x+y) = V^-(f(x+y)) = V^-(f(x)+f(y)) \leq \max\{V^-(f(x)), V^-(f(y))\} = \max\{A^-(x), A^-(y)\}$ which implies that $A^-(x+y) \leq \max\{A^-(x), A^-(y)\}$. And $A^-(xy) = V^-(f(xy)) = V^-(f(x)f(y)) \leq \max\{V^-(f(x)), V^-(f(y))\} = \max\{A^-(x), A^-(y)\}$ which implies that $A^-(xy) \leq \max\{A^-(x), A^-(y)\}$. Hence A is a bipolar valued fuzzy subsemiring of R.

**2.3 Theorem:** Let R and $R^l$ be any two semirings. The antihomomorphic image of a bipolar valued fuzzy subsemiring of R is a bipolar valued fuzzy subsemiring of $R^l$.

**Proof:** Let f: R → $R^l$ be an antihomomorphism. Let V = f(A) where A is a bipolar valued fuzzy subsemiring of R. We have to prove that V is a bipolar valued fuzzy subsemiring of $R^l$. Now for f(x), f(y) in $R^l$, $V^+(f(x)+f(y)) = V^+(f(y+x)) \geq A^+(y+x) \geq \min\{A^+(x), A^+(y)\} = \min\{V^+(f(x)), V^+(f(y))\}$ which implies that $V^+(f(x)+f(y)) \geq \min\{V^+(f(x)), V^+(f(y))\}$. And $V^+(f(x)f(y)) = V^+(f(yx)) \geq A^+(yx) \geq \min\{A^+(x), A^+(y)\} = \min\{V^+(f(x)), V^+(f(y))\}$ which implies that $V^+(f(x)f(y)) \geq \min\{V^+(f(x)), V^+(f(y))\}$. Also $V^-(f(x)+f(y)) = V^-(f(y+x)) \leq A^-(y+x) \leq \max\{A^-(x), A^-(y)\} = \max\{V^-(f(x)), V^-(f(y))\}$ which implies that $V^-(f(x)+f(y)) \leq \max\{V^-(f(x)), V^-(f(y))\}$. And $V^-(f(x)f(y)) = V^-(f(yx)) \leq A^-(yx) \leq \max\{A^-(x), A^-(y)\} = \max\{V^-(f(x)), V^-(f(y))\}$ which implies that $V^-(f(x)f(y)) \leq \max\{V^-(f(x)), V^-(f(y))\}$. Hence V is a bipolar valued fuzzy subsemiring of $R^l$.

 

**2.4 Theorem:** Let R and $R^{l}$ be any two semirings. The antihomomorphic preimage of a bipolar valued fuzzy subsemiring of $R^{l}$ is a bipolar valued fuzzy subsemiring of R.

**Proof:** Let f: $R \to R^{l}$ be an antihomomorphism. Let V = f(A) where V is a bipolar valued fuzzy subsemiring of $R^{l}$. We have to prove that A is a bipolar valued fuzzy subsemiring of R. Let x and y in R. Now $A^+(x+y) = V^+(f(x+y)) = V^+(f(y)+f(x)) \geq \min\{V^+(f(x)), V^+(f(y))\} = \min\{A^+(x), A^+(y)\}$ which implies that $A^+(x+y) \geq \min\{A^+(x), A^+(y)\}$. And $A^+(xy) = V^+(f(xy)) = V^+(f(y)f(x)) \geq \min\{V^+(f(x)), V^+(f(y))\} = \min\{A^+(x), A^+(y)\}$ which implies that $A^+(xy) \geq \min\{A^+(x), A^+(y)\}$. Also $A^-(x+y) = V^-(f(x+y)) = V^-(f(y)+f(x)) \leq \max\{V^-(f(x)), V^-(f(y))\} = \max\{A^-(x), A^-(y)\}$ which implies that $A^-(x+y) \leq \max\{A^-(x), A^-(y)\}$. And $A^-(xy) = V^-(f(xy)) = V^-(f(y)f(x)) \leq \max\{V^-(f(x)), V^-(f(y))\} = \max\{A^-(x), A^-(y)\}$ which implies that $A^-(xy) \leq \max\{A^-(x), A^-(y)\}$. Hence A is a bipolar valued fuzzy subsemiring of R.

**2.5 Theorem:** Let R and $R^{l}$ be any two semirings. The homomorphic image of a bipolar valued fuzzy normal subsemiring of R is a bipolar valued fuzzy normal subsemiring of $R^{l}$.

**Proof:** Let f: $R \to R^{l}$ be a homomorphism. Let V = f(A) where A is a bipolar valued fuzzy normal subsemiring of R. We have to prove that V is a bipolar valued fuzzy normal subsemiring of $R^{l}$. Now for f(x), f(y) in $R^{l}$, $V^+(f(x)+f(y)) = V^+(f(x+y)) \geq A^+(x+y) = A^+(y+x) \leq V^+(f(y+x)) = V^+(f(y)+f(x))$ which implies that $V^+(f(x)+f(y)) = V^+(f(y)+f(x))$. And $V^+(f(x)f(y)) = V^+(f(xy)) \geq A^+(xy) = A^+(yx) \leq V^+(f(yx)) = V^+(f(y)f(x))$ which implies that $V^+(f(x)f(y)) = V^+(f(y)f(x))$. Also $V^-(f(x)+f(y)) = V^-(f(x+y)) \geq A^-(x+y) = A^-(y+x) \leq V^-(f(y+x)) = V^-(f(y)+f(x))$ which implies that $V^-(f(x)+f(y)) = V^-(f(y)+f(x))$. And $V^-(f(x)f(y)) = V^-(f(xy)) \geq A^-(xy) = A^-(yx) \leq V^-(f(yx)) = V^-(f(y)f(x))$ which implies that $V^-(f(x)f(y)) = V^-(f(y)f(x))$. Hence V is a bipolar valued fuzzy normal subsemiring of $R^{l}$.

**2.6 Theorem:** Let R and $R^{l}$ be any two semirings. The homomorphic preimage of a bipolar valued fuzzy normal subsemiring of $R^{l}$ is a bipolar valued fuzzy normal subsemiring of R.

**Proof:** Let f: $R \to R^{l}$ be a homomorphism. Let V = f(A) where V is a bipolar valued fuzzy normal subsemiring of $R^{l}$. We have to prove that A is a bipolar valued fuzzy normal subsemiring of R. Let x and y in R. Now $A^+(x+y) = V^+(f(x+y)) = V^+(f(x)+f(y)) = V^+(f(y)+f(x)) = V^+(f(y+x)) = A^+(y+x)$ which implies that $A^+(x+y) = A^+(y+x)$. And $A^+(xy) = V^+(f(xy)) = V^+(f(x)f(y)) = V^+(f(y)f(x)) = V^+(f(yx)) = A^+(yx)$ which implies that $A^+(xy) = A^+(yx)$. Also $A^-(x+y) = V^-(f(x+y)) = V^-(f(x)+f(y)) = V^-(f(y)+f(x)) = V^-(f(y+x)) = A^-(y+x)$ which implies that $A^-(x+y) = A^-(y+x)$. And $A^-(xy) = V^-(f(xy)) = V^-(f(x)f(y)) = V^-(f(y)f(x)) = V^-(f(yx)) = A^-(yx)$ which implies that $A^-(xy) = A^-(yx)$. Hence A is a bipolar valued fuzzy normal subsemiring of R.

**2.7 Theorem:** Let R and $R^{l}$ be any two semirings. The antihomomorphic image of a bipolar valued fuzzy normal subsemiring of R is a bipolar valued fuzzy normal subsemiring of $R^{l}$.

**Proof:** Let f: $R \to R^{l}$ be an antihomomorphism. Let V = f(A) where A is a bipolar valued fuzzy normal subsemiring of R. We have to prove that V is a bipolar valued fuzzy normal subsemiring of $R^{l}$. Now for f(x), f(y) in $G^{l}$, $V^+(f(x)+f(y)) = V^+(f(y+x)) \geq A^+(y+x) = A^+(x+y) \leq V^+(f(x+y)) = V^+(f(y)+f(x))$ which implies that $V^+(f(x)+f(y)) = V^+(f(y)+f(x))$. And $V^+(f(x)f(y)) = V^+(f(yx)) \geq A^+(yx) = A^+(xy) \leq V^+(f(xy)) = V^+(f(y)f(x))$ which implies that $V^+(f(x)f(y)) = V^+(f(y)f(x))$. Also $V^-(f(x)+f(y)) = V^-(f(y+x)) \leq A^-(y+x) = A^-(x+y) \geq V^-(f(x+y)) = V^-(f(y)+f(x))$ which implies that $V^-(f(x)+f(y)) = V^-(f(y)+f(x))$. And $V^-(f(x)f(y)) = V^-(f(yx)) \leq A^-(yx) = A^-(xy) \geq V^-(f(xy)) = V^-(f(y)f(x))$ which implies that $V^-(f(x)f(y)) = V^-(f(y)f(x))$. Hence V is a bipolar valued fuzzy normal subsemiring of $R^{l}$.

**2.8 Theorem:** Let R and $R^{l}$ be any two semirings. The antihomomorphic preimage of a bipolar valued fuzzy normal subsemiring of $R^{l}$ is a bipolar valued fuzzy normal subsemiring of R.

**Proof:** Let f: $R \to R^{l}$ be an antihomomorphism. Let V = f(A) where V is a bipolar valued fuzzy normal subsemiring of $R^{l}$. We have to prove that A is a bipolar valued fuzzy normal subsemiring of R. Let x and y in R. Now $A^+(x+y) = V^+(f(x+y)) = V^+(f(y)+f(x)) = V^+(f(x)+f(y)) = V^+(f(y+x)) = A^+(y+x)$ which implies that $A^+(x+y) = A^+(y+x)$. And $A^+(xy) = V^+(f(xy)) = V^+(f(y)f(x)) = V^+(f(x)f(y)) = V^+(f(yx)) = A^+(yx)$ which implies that $A^+(xy) = A^+(yx)$. Also $A^-(x+y) = V^-(f(x+y)) = V^-(f(y)+f(x)) = V^-(f(x)+f(y)) = V^-(f(y+x)) = A^-(y+x)$ which implies that $A^-(x+y) = A^-(y+x)$. And $A^-(xy) = V^-(f(xy)) = V^-(f(y)f(x)) = V^-(f(x)f(y)) = V^-(f(yx)) = A^-(yx)$ which implies that $A^-(xy) = A^-(yx)$. Hence A is a bipolar valued fuzzy normal subsemiring of R.

## REFERENCES

1. Anitha.M.S., Murugnantha Prasad & K.Arjunan, Notes on Bipolar-valued fuzzy subgroups of a group, Bulletin of Society for Mathematical Services and Standards, Vol. 2 No. 3 (2013), pp. 52-59.
2. Anitha.M.S, K.L.Muruganantha Prasad & K.Arjunan, Homomorphism and anti-homomorphism of Bipolar valued fuzzy subgroups of a group, International Journal of Mathematical Archive-4(12), 2013, 1-4.
3. Anthony.J.M and H.Sherwood, fuzzy groups Redefined, Journal of mathematical analysis and applications, 69(1979), 124 -130.
4. Arsham Borumand Saeid, Bipolar-valued fuzzy BCK/BCI-algebras, World Applied Sciences Journal 7 (11) (2009), 1404-1411.
5. Azriel Rosenfeld, fuzzy groups, Journal of mathematical analysis and applications 35(1971), 512-517.
6. F.P.Choudhury, A.B.Chakraborty and S.S.Khare, A note on fuzzy subgroups and fuzzy homomorphism, Journal of mathematical analysis and applications, 131(1988), 537 -553.
7. W.L.Gau and D.J. Buehrer, Vague sets, IEEE Transactons on Systems, Man and Cybernetics, 23(1993), 610-614.
8. Kyoung Ja Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull. Malays.Math. Sci. Soc. (2) 32(3) (2009), 361–373.
9. K.M.Lee, Bipolar-valued fuzzy sets and their operations. Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, (2000), 307-312.
10. K.M.Lee, Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolarvalued fuzzy sets. J. fuzzy Logic Intelligent Systems, 14 (2) (2004), 125-129.
11. Mustafa Akgul, some properties of fuzzy groups, Jurnal of mathematical analysis and applications, 133(1988), 93 -100.
12. Samit Kumar Majumder, Bipolar valued fuzzy sets in Γ-semigroups, Mathematica Aeterna, Vol. 2, no. 3(2012), 203 – 213.
13. Young Bae Jun and Seok Zun Song, Subalgebras and closed ideals of BCH-algebras based on bipolar-valued fuzzy sets, Scientiae Mathematicae Japonicae Online, (2008), 427-437.
14. L.A.Zadeh, fuzzy sets, Inform. And Control, 8(1965), 338-353.

**Source of support: Nil, Conflict of interest: None Declared**