RESTRICTIONS OF PRE A*-ALGEBRA FUNCTIONS

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(Received On: 17-11-18; Revised & Accepted On: 12-12-18)

ABSTRACT
In this paper restriction of Pre A*-algebra function has been derived. Shannon expansion of Pre A*-algebra function is explained with an example. Theorems related to the restriction have been proved.

Key words: Restriction of Pre A*-algebra function, Shannon expansion

1. INTRODUCTION
In 1994, P. Koteswara Rao [1] first introduced the concept of A*-algebra \( (A, \wedge, \vee, (-)^{-}) \).
In 2000, J. Venkateswara Rao [2] introduced the concept Pre A*-algebra \( (A, \wedge, \vee, (-)^{-}) \) analogous to C-algebra as a reduct of A*-algebra. In [4] ternary operation on Pre-A* algebra have been proved and studied the properties. J. Venkateswara Rao [5] analyze the properties of PreA*-function. He defined implicants of Pre A*-algebra function[6].

2. PRELIMINARIES
Definition 2.1 [4]: An algebra \( (A, \wedge, \vee, (-)^{-}) \) where A is non-empty set with \( 1, \wedge, \vee \) are binary operations and \( (-)^{-} \) is a unary operation satisfying

(a) \( x^{-} = x \), \( \forall x \in A \)
(b) \( x \wedge x = x \), \( \forall x \in A \)
(c) \( x \wedge y = y \wedge x \), \( \forall x, y \in A \)
(d) \( (x \wedge y)^{-} = x^{-} \vee y^{-} \), \( \forall x, y \in A \)
(e) \( x \wedge (y \wedge z) = (x \wedge y) \wedge z \), \( \forall x, y, z \in A \)
(f) \( x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) \), \( \forall x, y, z \in A \)
(g) \( x \wedge y = x \wedge (x^{-} \vee y^{-}) \), \( \forall x, y \in A \).

is called a Pre A*-algebra

Example 2.1[4]: \( 3 = \{0, 1, 2\} \) with operations \( \wedge, \vee, (-)^{-} \) defined below is a Pre A*-algebra.

\[
\begin{array}{c|c|c|c|c|c|c|c}
\wedge & 0 & 1 & 2 & \vee & 0 & 1 & 2 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \\
1 & 0 & 1 & 2 & 1 & 1 & 1 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
\end{array}
\]

x | x^{-} | 0 | 1 | 2
---|---|---|---|---
0 | 0 | 1 | 1 | 0
1 | 1 | 0 | 0 | 1
2 | 2 | 2 | 2 | 2

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Lemma 2.2 [4]: Every Pre A*-algebra with 1 satisfies the following laws
(a) \( x \lor 1 = x \lor x' \)  
(b) \( x \land 0 = x \land x' \)

Lemma 2.3 [4]: Every Pre A*-algebra with 1 satisfies the following laws.
(a) \( x \land (x' \lor x) = x \lor x' \)
(b) \( (x \lor x') \land y = (x \land y) \lor (x' \land y) \)
(c) \( (x \lor y) \land z = (x \land z) \lor (x' \land y \land z) \)

Definition 2.4 [4]: Let \( A \) be a Pre A*-algebra. An element \( x \in A \) is called central element of \( A \) if \( x \land x' = 1 \) and the set \( \{ x \in A / x \land x' = 1 \} \) of all central elements of \( A \) is called the centre of \( A \) and it is denoted by \( B(A) \).

Theorem 2.5 [4]: Let \( A \) be a Pre A*-algebra with 1, then \( B(A) \) is a Boolean algebra with the induced operations \( \land, \lor, (\cdot)' \)

Theorem 2.6 [4]: Let \( A \) is a Pre A*-algebra with 1. Then \( A \) has trivial centre if and only if \( A = \overline{A_o} \), for some Pre A*-algebra \( A_o \).

Lemma 2.7 [4]: Let \( A \) be a Pre A*-algebra with 1, 
(a) If \( y \in B(A) \) then \( x \land x' \land y = x \lor x' \land \forall x \in A \)
(b) If \( x, y \in B(A) \) then \( x \land (x \lor y) = x \lor (x \land y) = x \)

Lemma 2.8 [4]: Let \( A \) be a Pre A*algebra with 1, 0 and let \( x, y \in A \)
(a) If \( x \lor y = 0 \), then \( x = y = 0 \)  
(b) If \( x \lor y = 1 \), then \( x \lor x' = 1 \)

Theorem 2.9 [4]: Let \( A \) be a Pre A*-algebra with 1 and \( x, y \in A \), if \( x \land y = 0 \), \( x \lor y = 1 \), then \( y = x' \)

Definition 2.10[7]: A Pre A* -algebra function is said to be in disjunctive normal form in \( n \) variables \( x_1, x_2, x_3, \ldots, x_n \) if it can be written as join of terms of the type \( f_1(x_1) \land f_2(x_2) \land \ldots \land f_n(x_n) \) where \( f_i(x_i) = x_i' \) or \( x_i \) \( \forall i = 1 \) to \( n \) and no two terms are same. \( f_i(x_i) \land f_2(x_2) \land \ldots \land f_n(x_n) \) are called minterms or minimal polynomials.
Thus a minterm in \( n \) variables is a product of \( n \) literals in which each variable is represented by the variable itself or its complement.

Definition 2.11[7]: If a DNF contains all the possible minterms then it is complete DNF.

Definition 2.12[7]: A Pre A* -algebra function is said to be in conjunctive normal form in \( n \) variables \( x_1, x_2, x_3, \ldots, x_n \) if it can be written as meet of terms of the type \( f_1(x_1) \lor f_2(x_2) \lor \ldots \lor f_n(x_n) \) where \( f_i(x_i) = x_i \) or \( x_i' \) \( \forall i = 1 \) to \( n \) and no two terms are same. \( f_1(x_1) \lor f_2(x_2) \lor \ldots \lor f_n(x_n) \) are called maxterms or maximal polynomials

3. RESTRICTION OF PRE A*-ALGEBRA FUNCTION

If \( X_1 \) is any subset of \( X \), the restriction of function is the function \( f_{|X_1} \) from \( X_1 \) to \( Y \).

If \( f_{|X_1} \) is the restriction of \( f \), then \( f \) is the extension of \( f_{|X_1} \). Informally, a restriction of a function \( f \) is the result of trimming its domain.

Definition 3.1: Let \( f \) be a Pre A* -function on \( A^* \) and let \( k \in \{1, 2, \ldots, n\} \). We denote by \( f_{|X_1} = 2, f_{|X_1} = 1, \) and \( f_{|X_1} = 0 \) respectively, the Pre A* -function defined as follows:
for every \( (\alpha_1, \alpha_2, \ldots, \alpha_{k-1}, \alpha_{k+1}, \ldots, \alpha_n) \in A^{n-1} \)
\( f_{|X_1} (\alpha_1, \alpha_2, \ldots, \alpha_{k-1}, \alpha_{k+1}, \ldots, \alpha_n) = f(2) \)
\[ f_{k=1}(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1}, \ldots \alpha_n) = f(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, 1, \alpha_{k+1}, \ldots \alpha_n) \]
\[ f_{k=0}(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1}, \ldots \alpha_n) = f(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, 0, \alpha_{k+1}, \ldots \alpha_n) \]

\[ f_{k=2} \] is the restriction of \( f \) to \( f(2) \)
\[ f_{k=1} \] is the restriction of \( f \) to \( f(1, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1}, \ldots \alpha_n) \) in which \( \alpha_k = 1 \)
\[ f_{k=0} \] is the restriction of \( f \) to \( f(0, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1}, \ldots \alpha_n) \) in which \( \alpha_k = 0 \)

Even though \( f_{k=2}, f_{k=1}, \text{and } f_{k=0} \) are by definition, functions of \((n-1)\) variables, it is considered as functions on \( A^n \) rather than \( A^{n-1} \) for every \( (\alpha_1, \alpha_2, \ldots \alpha_n) \in A^n \), we simply let
\[ f_{k=2}(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1}, \ldots \alpha_n) = f(2) \]
\[ f_{k=1}(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1}, \ldots \alpha_n) = f(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, 1, \alpha_{k+1}, \ldots \alpha_n) \]
\[ f_{k=0}(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1}, \ldots \alpha_n) = f(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, 0, \alpha_{k+1}, \ldots \alpha_n) \]

**Theorem 3.2:** Let \( f \) be a Pre A*-function on \( A^n \). Let \( \psi \) be a representation of \( f \) and let \( k \in \{1, 2, \ldots, n\} \) Then the expression obtained by substituting the constant 0 or 1 or 2 for every occurrence of \( x_k \) in \( \psi \) represents \( f_{k=0} \) or \( f_{k=1} \) or \( f_{k=2} \).

**Proof:** This is an immediate consequence of above definition.

**Example 3.3:** Consider Pre A*-function
\[ f = (\alpha \land \beta) \lor (\alpha \land \gamma) \lor (\beta \land \gamma) \]

We derive the following expressions for \( f_{k=2}, f_{k=1}, \text{and } f_{k=0} \)
\[ f = (\alpha \land \beta) \lor (\alpha \land \gamma) \lor (\beta \land \gamma) \]
\[ f_{k=2} = (2 \land \beta) \lor (2 \land \gamma) \lor (\beta \land \gamma) \]
\[ f_{k=1} = (1 \land \beta) \lor (1 \land \gamma) \lor (\beta \land \gamma) \]
\[ f_{k=0} = (0 \land \beta) \lor (0 \land \gamma) \lor (\beta \land \gamma) \]

**Theorem 3.4:** Let \( f \) be a Pre A*-function on \( A^n \) and let \( k \in \{1, 2, \ldots, n\} \).

Then \( f(\alpha_1, \alpha_2, \ldots \alpha_n) \equiv \alpha_k f_{k=2} \lor \alpha_k \neg f_{k=2} \lor \alpha_k f_{k=1} \lor \alpha_k \neg f_{k=1} \lor \alpha_k f_{k=0} \lor \alpha_k \neg f_{k=0} \) for all \( (\alpha_1, \alpha_2, \ldots \alpha_n) \in A^n \).

**Proof:** This is immediate by substitute of the values \( \alpha_k = 2, \alpha_k = 1, \text{or } \alpha_k = 0 \)
\[ f(\alpha_1, \alpha_2, \ldots \alpha_n) = 2 f_{k=2} \]
\[ f(\alpha_1, \alpha_2, \ldots 0, \ldots \alpha_n) = f_{k=1} \]
\[ f(\alpha_1, \alpha_2, \ldots 0. \alpha_n) = 0 f_{k=0} \]
\[ f(\alpha_1, \alpha_2, \ldots \alpha_n) = 2 f_{k=2} \lor 2 f_{k=1} \lor 1 f_{k=1} \lor 0 f_{k=0} \]

**Example 3.5:** Consider the function \( f = (\alpha \land \beta) \lor (\alpha \land \gamma) \lor (\alpha \land \beta) \lor (\beta \land \gamma) \)
The expansion of \( f \) with respect to \( \alpha \) is \( \alpha f_{\beta=1 \alpha=1} \lor \alpha \neg f_{\beta=1 \alpha=0} \lor \alpha f_{\beta=1 \alpha=2} \lor \alpha \neg f_{\beta=1 \alpha=2} \)
\[ f = (\alpha \land \beta) \lor (\alpha \land \gamma) \lor (\alpha \land \beta) \lor (\beta \land \gamma) \]
The expansion of \( f_{\beta=1} \) with respect to \( \alpha \) is

\[
\alpha f_{\beta=1, \alpha=1} \lor \alpha^c f_{\beta=1, \alpha=0} \lor \alpha f_{\beta=1, \alpha=0} \lor \alpha^c f_{\beta=1, \alpha=2} = 1(1) \lor 0(0) \lor 2(2) = 2
\]

The expansion of \( f_{\beta=0} \) with respect to \( \alpha \) is

\[
\alpha f_{\beta=0, \alpha=1} \lor \alpha^c f_{\beta=0, \alpha=0} \lor \alpha f_{\beta=0, \alpha=0} \lor \alpha^c f_{\beta=0, \alpha=2} = 1(1) \lor 0(0) \lor 2(2) = 2
\]

The expansion of \( f_{\beta=2} \) with respect to \( \alpha \) is

\[
\alpha f_{\beta=2, \alpha=1} \lor \alpha^c f_{\beta=2, \alpha=0} \lor \alpha f_{\beta=2, \alpha=0} \lor \alpha^c f_{\beta=2, \alpha=2} = 1(1) \lor 0(0) \lor 2(2) = 2
\]

Similarly, we can write the expansion for \( \beta = 2 \) with respect to \( \gamma \).

Note 3.6: The expansion \( \alpha f_{\beta=1, \alpha=1} \lor \alpha^c f_{\beta=1, \alpha=0} \lor \alpha f_{\beta=1, \alpha=0} \lor \alpha^c f_{\beta=1, \alpha=2} \) is called as Shannon expansion. By applying this expansion to a function and its restriction becomes 0 or 1 or 2 or a literal.

REFERENCES


Source of support: Nil, Conflict of interest: None Declared.

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