PART I KALANGI NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACE OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD

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ABSTRACT

In this manuscript we introduce new notions on PART I Kalangi non-associated Γ-semi sub near-field space of a Γ-near-field space over near-field, quasi non associative Γ-semi sub near-field space, K-quasi N -Γ-semi sub near-field space, quasi ideals, etc and concepts like PART I Kalangi quasi bipotent elements and several analogous properties done in case of Γ-near-field spaces.

Keywords: Non-associative Γ-semi sub near-field space, Kalangi-Γ-semi sub near-field space, Γ-semi sub near-field space of Γ-near-field space; Semi near-field space of Γ-near-field space, quasi Γ-semi sub near-field space, quasi non-associative Γ-semi near-field space.


SECTION 1: INTRODUCTION AND PRELIMINARIES

In this paper we together introduced several concepts and new notions in PART I Kalangi non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field like quasi non associative Γ-semi sub near-field space, K-quasi N -Γ-semi sub near-field space, quasi ideals, etc and concepts like PART I Kalinga quasi bipotent elements and several analogous properties done in case of Γ-near-field spaces.

Definition 1.1: Let N be a K-quasi non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field an element x is said to be quasi central if xy = yx for all y ∈ M; M ⊆ N is a Γ-near-field (or M ⊆ N and M is a Γ-semi near-field space).

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Definition 1.2: Let $N$ be a $K$-quasi non-associative $\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field we say $N$ is said to be Kalinga quasi non-associative sub-directly irreducible $\Gamma$-semi sub near-field space ($K$-quasi sub-directly irreducible non-associative $\Gamma$-semi sub near-field space) if the intersection of all non zero $K$-quasi ideals of $N$ is non-zero.

Definition 1.3: Let $N$ be a $K$-quasi non-associative $\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field we say $N$ is said to have Kalinga quasi intersection of factors property ($K$-quasi IFP) if $a, b \in N$, $ab = 0$ implies $amb = 0$ where $m \in M$, $M \subset N$ and $M$ is a near-field (or $m \in M$, $M \subset N$, $M$ is a non-associative $\Gamma$-semi sub near-field space.

Note 1.4: We can take $(nb) = (an)$ $b$ in all cases it should vanish that is $anb = 0$.

Now we define the concept of Kalangi quasi divisibility and Kalangi divisibility.

Definition 1.5: Let $N$ be a non-associative $K$-$\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field we say $N$ is Kalangi weakly divisible ($K$-weakly divisible) if for all $x, y \in N$ there exists a $z \in P$; $P \subset N$ where $P$ is an associative $\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field or $P$ is a near-field such that $xz = y$ or $zx = y$.

Definition 1.6: Let $N$ be a non-associative $K$-$\Gamma$-semi near-field space of a $\Gamma$-near-field space over near-field we say $N$ is Kalangi weakly divisible ($K$-weakly divisible) if for all $x, y \in N$ there exists a $z \in P$; $P \subset N$ where $P$ is an associative $\Gamma$-semi near-field space of a $\Gamma$-near-field space over near-field or $P$ is a near-field such that $xz = y$ or $zx = y$.

Definition 1.7: Let $N$ be a $K$-quasi sub near-field space ($K$-$\Gamma$-semi sub near-field spaces) we say $N$ is said to be a Kalinga strongly prime ($K$-strongly prime) $I$ if for each $a \in N \setminus \{0\}$ there exists a finite $K$-$\Gamma$-semi sub near-field space $F$ such that $Fx \neq 0$ for all $x \in P \setminus \{0\}$, $P \subset N$; $P$ is a associative $\Gamma$-semi sub near-field space/ $P$ is a associative $\Gamma$-semi near-field space.

In case $N$ is $K$-$\Gamma$-semi sub near-field space $II$ (III or IV) we say $N$ is a Kalinga strong prime $II$ (III or IV) ($K$-strong prime $II$(III or IV)) if for each $a \in N \setminus \{0\}$ there exists a finite $K$-$\Gamma$-semi sub near-field space $F$ such that $Fx \neq 0$ for all $x \in P \setminus \{0\}$, $P \subset N$; $P$ is a associative $\Gamma$-semi sub near-field space/ $P$ is a associative $\Gamma$-semi near-field space.

Definition 1.9: Let $N$ be a non associative right near-field space and $A$ an $K$-ideal or a $K$-left ideal of $N$, we define three properties as follows

(i) $A$ is kalangi equi-prime ($K$-equi-prime) if for any $a, x, y \in N$ such that $a (nx) – a(ny) \in A \forall \in N$ or

$(an)x – (an)y \in Y$ we have $a \in A$ or $x – y \in A$.

(ii) $A$ is Kalangi strongly semi prime ($K$-strongly semi prime) if for each a finite subset $F$ of $N$ such that if $x, y \in N$ and ($af$) $x – (af) y \in A$ or

$y \in A$ and $a (fx) – a (fy) \in A$ or

$(af) x – a (fy) \in A$ or

$A$ is a $K$-ideal or a $K$-left ideal of $N$ such that $x – y \in A$.

(iii) $A$ is Kalangi completely equi prime ($K$-completely equi prime) if $a \in N \setminus A$ and $ax – ay \in A$ imply $x – y \in A$.

Definition 1.10: Let $Q$ be a non empty subset of a $K$-right $\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field $N$ which is non-associative. Define left and right Kalangi polar subsets ($K$-polar subsets) of $N$ by

$SL(Q) = \{x \in Q | x(NQ) = 0 \}$ or

and $SR(Q) = \{y / (qN) y = 0 \}$ or

$q(Ny) = 0$ for all $q \in Q$.

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Suppose, $SQ_L(N)$ is the set of $Q$-left polar subsets of $N$ and $SQ_R(N)$ is the set of $Q$-right polar subsets of $N$ one need to test whether $SQ_L(N)$ and $SQ_R(N)$ are complete bounded lattices.

Definition 1.11: $I, II$ and $III$ three levels of Kalangi $\Gamma$-semi sub pseudo near-field space($K$-$\Gamma$-SSPNFS). Let $Q$ be a $\Gamma$-semi sub pseudo near-field space ($\Gamma$-SSPNFS) of a $\Gamma$-near-field space over near-field $N$ we say $Q$ is a Kalinga $\Gamma$-SSPNFS $I$ ($K$-$\Gamma$-SSPNFS $I$) if $Q$ has a proper subset $T \subset Q$ such that $T$ is a $\Gamma$-semi sub near-field space. Kalangi $\Gamma$-semi sub pseudo near-field space $II$ ($K$-$\Gamma$-SSPNFS $II$) if $Q$ has proper subset $M \subset Q$ such that $M$ is a $\Gamma$-semi sub near-field space. Kalangi $\Gamma$-semi sub pseudo near-field space $III$ ($K$-$\Gamma$-SSPNFS $III$) if $Q$ has a proper $W \subset Q$.

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such that \((W, \oplus, \otimes)\) is a \(\Gamma\)-semi near-field space. Thus we have three levels (I, II and III) of \(K\)-\(\Gamma\)-SSPNFS near-field spaces over a near-field \(N\). A Kalangi \(\Gamma\)-SSPNFS \(\Gamma\)-semi near-field space (\(K\)-\(\Gamma\)-SSPNFS) is defined as a proper subset \(U\) of \(Q\) such that \((U, \oplus, \otimes)\) is a \(K\)-\(\Gamma\)-SSPNFS \(\Gamma\)-semi near-field space.

**Definition 1.12:** Let \((Q, \oplus, \otimes)\) be a \(\Gamma\)-semi sub pseudo near-field space (\(\Gamma\)-SSPNFS) of a \(\Gamma\)-near-field space over near-field. A proper subset \(I\) of \(Q\) is called a Kalangi \(\Gamma\)-semi sub pseudo near-field space ideal (\(K\)-\(\Gamma\)-SSPNFS-ideal) if

- a. for all \(p, q \in I, p \oplus q \in I\)
- b. \(0 \in I\)
- c. for all \(p \in I\) and \(r \in P\) we have \(p \otimes r \in I\) or \(r \otimes p \in I\).
- d. \(I\) is a \(K\)-\(\Gamma\)-SSPNFS, \(\Gamma\)-semi near-field space.

**Definition 1.13:** Let \((N, \oplus, \otimes)\) and \((N_1, \oplus, \otimes)\) be any two \(K\)-\(\Gamma\)-semi sub pseudo near-field spaces (\(\Gamma\)-SSPNFS) of a \(\Gamma\)-near-field space over near-field. \(M\) is said to be a Kalangi quasi \(\Gamma\)-semi sub pseudo near-field space (\(K\)-\(\Gamma\)-SSPNFS) if and only if \(M\) is a \(K\)-\(\Gamma\)-SSPNFS \(\Gamma\)-semi near-field space.

**Definition 1.14:** Let \((N, \oplus, \otimes)\) be a \(\Gamma\)-semi sub pseudo near-field space (\(\Gamma\)-SSPNFS) of a \(\Gamma\)-near-field space over near-field. \(M\) is said to be a Kalangi quasi \(\Gamma\)-semi sub pseudo near-field space (\(K\)-\(\Gamma\)-SSPNFS) if and only if \(M\) is a \(K\)-\(\Gamma\)-SSPNFS \(\Gamma\)-semi near-field space.

**SECTION 2: MAIN RESULT ON KALANGI \(-\)QUASI GAMMA SEMI PSEUDO SUB NEAR-FIELD SPACES OF A GAMMA NEAR-FIELD SPACE OVER A NEAR-FIELD.**

In this section, author present theorem as main result on Kalangi quasi Gamma semi pseudo sub near-field spaces of a Gamma near-field space over near-field.

Now we proceed on to define Kalangi right quasi regular element. We just recall that an element \(x \in N\), \(N\) is a Gamma semi pseudo sub near-field space said to be the right quasi regular if there exist \(y \in N\) such that \(x \circ y = x + y - xy = 0\) and left quasi regular if there exist \(y \in N\) such that \(y \circ x = 0 = y' + x - yx\).

The study of the quasi regular concept happens to be an interesting study in case of near-field spaces and semi near-field spaces.

Quasi regular if it is right and left quasi regular simultaneously. We say an element \(x \in N\) is Kalangi right quasi regular (K-right quasi regular) if there exist \(y\) and \(z \in N\) such that \(x \circ y = x + y - xy = 0\) and \(z \circ y = y + z - yz = 0\) but \(y \circ z = y + z - yz \neq 0\) and \(z \circ y = y + z - yz \neq 0\).

Similarly we define Kalangi left quasi regular (K-left quasi regular) and \(x\) will be Kalangi quasi regular (K-quasi regular) if it is simultaneously K-right quasi regular and K-left quasi regular, that is of Kalangi quasi Gamma semi pseudo sub near-field spaces of a Gamma near-field space over a near-field.

If we define K-non-associative \(\Gamma\)-semi pseudo sub near-field space of a \(\Gamma\)-near-field space over near-field (K-quasi \(\Gamma\)-semi pseudo sub near-field space) \(N\) then we have main interesting result out of several results below.

**Theorem 2.1:** Let \(N\) be a Kalangi quasi \(-\)\(\Gamma\)-semi pseudo sub near-field space of a \(\Gamma\)-near-field space over near-field (K-quasi \(\Gamma\)-semi pseudo sub near-field space) having a proper subset \(P\) of \(N\) to be a commutative near-field space with unit and of a characteristic 0. \(L\) any loop of finite order. Then the near loop near-field space \(NL\) has a right quasi regular element \(x = \sum \alpha_i m_i (m_i \in L) \alpha_i \in P \subset N\) is right quasi regular then \(\sum \alpha_i \neq 1\).

**Proof:** Let \(y = \sum \beta_i h_i\) where \(\beta_i \in P\) and \(h_i \in L\) be the right quasi inverse of \(x\) then \(x + y - xy = 0\)

\[\text{i.e., } \sum \alpha_i m_i - \sum \beta_i h_i - (xy) = 0.\]

Equating the coefficients of the like terms and adding these coefficients we get,

\[\sum \alpha_i + \sum \beta_i = \sum \alpha_i \sum \beta_i = 0. \text{ or } \sum \alpha_i - \sum \beta_i = \sum \alpha_i \sum \beta_i = 0.\]

Now if \(\sum \alpha_i = 1\) then \(\sum \alpha_i = 0\) a contradiction. Hence \(\sum \alpha_i \neq 0\). This completes the proof of the theorem.

**Example 2.2:** Let \(L\) be any finite loop, \(N = Z_7 \times Z_7\) be K-mixed direct product of the Kalangi quasi \(-\)\(\Gamma\)-semi pseudo sub near-field space of a \(\Gamma\)-near-field space over near-field (K-quasi \(\Gamma\)-semi pseudo sub near-field space) \(Z_7\) and the prime field of characterize 7, \(Z_7\), \(N\) is K- quasi \(-\)\(\Gamma\)-semi pseudo sub near-field space, \(NL\) is the near loop near-field of the loop \(L\) over the near-field space \(N\). If \(x \in S (J (Z_7L))\).
Definition 2.3: Let $N = N_1 \times N_2$ where $N_i$ is a Kalangi quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space over near-field space of $N_i$ (K-quasi $\Gamma$-semi pseudo sub near-field space) of characterize $0$ and $N_2$ is any quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space over near-field of the near loop quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space over near-field of the loop $L$ over the quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space $N$.

Definition 2.4: $QJ(Q)$ said to be the Kalangi Jacobson radical (K-Jacobson radical) of $N$ if $Q \subseteq NL$ is a non-associative quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space and $J(Q)$ denoted the usual Jacobson radical of the non-associative quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space $Q$.

Example 2.5: Let $N = Z \times Z_{18}$ be the mixed direct product of the Kalangi quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space over near-field (K-quasi $\Gamma$-semi pseudo sub near-field space) $Z$ and the semi pseudo sub near-field space $Z_{18}$ any finite loop, $NL$ the near loop of the loop $L$ over the Kalangi quasi-$\Gamma$-semi pseudo sub near-field space $N$, clearly $ZL \subseteq NL$ and $ZL$ is a non-associative Kalangi quasi-$\Gamma$-semi pseudo sub near-field space $N$. If $x = \sum \alpha_i h_i \in ZL$ such that $\sum \alpha_i \neq 0$ then $x \notin QJ(ZL)$. It is left for the scholar or reader to verify, as the conclusion derived is straightforward.

Theorem 2.6: Let $N = Z_2 \times Z_{15}$ where $Z_2$ is the prime Kalangi quasi-$\Gamma$-semi pseudo sub near-field space of a $\Gamma$-near-field space over near-field (K-quasi $\Gamma$-semi pseudo sub near-field space) $Z$ and $Z_{15}$ is a $\Gamma$-semi pseudo sub near-field space over near-field. Let $L$ be any loop, $NL$ be the near loop Kalangi quasi-$\Gamma$-semi pseudo sub near-field space. If $x \in Z_2 L \times \{0\} \subseteq (Z_2 \times Z_{15})$; $L$ is right quasi regular Kalangi quasi-$\Gamma$-semi pseudo sub near-field space then $|\text{supp } x|$ is an even number.

Proof: The proof is obvious and easily obtained by simple calculations.

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