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STAR CHROMATIC NUMBER OF $C(H_n)$, $C(G_n)$, $L(H_n)$, $L(G_n)$, $L(W_n)$

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ABSTRACT

In this paper, we discuss about the star colouring and its chromatic number of the Line and Central graphs of Helm graph, Gear graph and Wheel graph H_n, G_n and W_n respectively.

Keywords: *Chromatic number, Central graph, Line graph, Star colouring, Star chromatic number.*

Subject Classification: *05C15, 05C75.*

1. INTRODUCTION

Let G be a finite undirected graph with no loops and multiple edges. The Line graph $L(G)$ of a graph $G = (V,E)$ is a graph with vertex set $E(G)$ in which two vertices are adjacent if and only if the corresponding edges in G are adjacent.

The Central graph of a graph G [7], $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G.

The Helm graph H_n [6] is the graph obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.

The Gear graph G_n [6] also known as a bipartite wheel graph, is a wheel graph with a graph vertex added between each pair of adjacent graph vertices of the outer cycle.

A Wheel graph W_n [6] of order n is a graph that contains a cycle of order $n-1$ and for which every graph vertex in the cycle is connected to another graph vertex (which is known as hub). The edges of a wheel which includes the hub are called spokes. The wheel can be defined as the graph K_1+C_{n-1}.

A Star colouring of a graph G [1, 5] is a proper vertex colouring (no two adjacent vertices of G have the same colour) such that every path of G on four vertices is not bicoloured. The minimum number of colours needed to star colour G is called as Star chromatic number and is denoted by $X_s(G)$.The Star chromatic number has been introduced by Grunbaum in 1973.

2. STAR COLOURING OF LINE GRAPH OF GEAR GRAPH $L(G_n)$

2.1 Theorem: For the Line graph of G_n, $X_s[L(G_n)] = n$ for $n \geq 3$.

Proof: Let G_n be a gear graph formed by the wheel graph W_n of n vertices. The edge set of G_n are given as follows. Let
(i) $\{e_1,e_2,\ldots e_n\}$ be the spokes of G_n.
(ii) $\{e_{n+1},e_{n+2},\ldots e_{3n}\}$ be the edges of the cycle in G_n.

Thus the vertex set of $L(G_n)$ is given as $\{v_1,v_2,v_3,\ldots\ldots v_{3n}\}$. The vertex subset $\{v_1, v_2,\ldots v_n\}$ forms a complete graph in $L(G_n)$. Now assign a star colouring as follows. Assign the colour c_i to the vertex v_i for $i=1, 2, 3\ldots n$. Next we have to colour the vertices $\{v_{n+1}, v_{n+2},\ldots v_{3n}\}$. Since the colouring should be minimum, we cannot introduce new colours to those vertices. So assign only the existing colours to those vertices.

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Next we assign the colour c_i for $i=1, 2,\ldots n$ to the vertices v_i for $i= n+1$ to$2n$. Assign the colours c_i for $i=1, 2\ldots n-2$ to the vertices $i=2n+1$ to $3n-2$ and the colours c_n to v_{3n-1} and c_{n-1} to v_{3n}. Clearly the colouring is minimum and a star colouring.

Example:

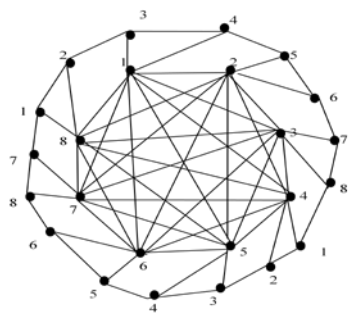

Figure: 1 $X_s\,[L\,(G_8)] =8$

3. STAR COLOURING OF CENTRAL GRAPH OF GEAR GRAPH $C(G_n)$

3.1 Theorem: For Central graph of H_n, $X_s[C\,(G_n)] = 2n+1$.

Proof: Let $\{v_1, v_2,\ldots\ldots v_n\}$ be the vertex set of the cycle and v be the hub. The hub and the vertices v_i and the newly introduced vertices vv_i for $i=1$ to n forms a $k_{1,2n}$ star graph. Now assign the star colouring as follows.

Assign the colour c_i to v_{2i-1} for $i=1$ to n and the colour c_{n+i} to vv_i for $i=1,2,3,\ldots\ldots n$. Also, colour the vertices c_{n+i} to v_{2i} for $i=1$ to n and the hub v is assigned the colour c_{2n+1}.

Every v_i , for $i=1$ to n has three adjacent vertices that is to be coloured with different colours. Since the colouring should be minimum, we try to repeat the colours that are already used.

Next we have to colour the v_{ij}'s , c_{i+1} to v_{ij} for $i=1$ to $n-1$, $j=n+i$ and the colour c_1 to v_{ij} for $i=n$, $j=2n$. Also the colour c_{n+i+1} for $i=1$ to n to the vertices v_{ij} for $i=n+i$,$j=2$ to $n-1$ and the colour c_{n+1} to v_{ij} for $i=2n$, $j=1$.

This is clearly star colouring, since the three adjacent vertices v_{ij} to the vertex v_i get different colours and the colouring is also minimum.

Thus, $X_s[C\,(H_n)] = 2n+1$.

Example:

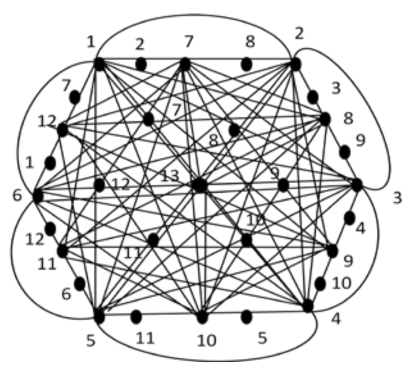

Figure: 2 $X_s[C\,(H_6)] =13$

4. STAR COLOURING OF LINE GRAPH OF HELM GRAPH $L(\mathrm{H_n})$

4.1 Theorem: For the Line graph of H_n, $X_s\,[L\,(H_n)] = n$, for all $n \geq 3$.

Proof: Let H_n be a helm graph obtained from adding a pendant edge at each vertex of the cycle. The edge set of H_n are given as follows: Let
(i) $\{e_1, e_2, e_3,\ldots e_n\}$be the spokes of H_n.
(ii) $\{e_{n+1}, e_{n+2}, e_{n+3}\ldots e_{2n}\}$ be the edges of the cycle in H_n.
(iii)$\{e_{2n+1}, e_{2n+2}\ldots. e_{3n}\}$ be the pendant edges.

Let the vertex set of $L(H_n)$ be $\{v_1,v_2,v_3,\ldots\ldots\ldots v_{3n}\}$.The vertex subset $\{v_1,v_2,v_3,\ldots\ldots\ldots v_n\}$ forms a complete graph. Now assign the star colouring as follows. Assign the colour c_i to the vertex v_i for $i=1, 2, 3\ldots.n$. Next we have to colour the vertices $\{v_{n+1}, v_{n+2,\ldots} v_{3n}\}$. Since the colouring should be minimum, we cannot introduce new colours to those vertices. So assign only the existing colours to those vertices.

Next we assign the colour c_i for $i=1, 2,\ldots n$ to the vertices v_i for $i= n+1$ to $2n$. Assign the colours c_i for $i=1, 2,\ldots n-2$ to the vertices $i=2n+1$ to $3n-2$ and the colours c_n to v_{3n-1} and c_{n-1} to v_{3n}. Clearly the colouring is minimum and a star colouring.

Example:

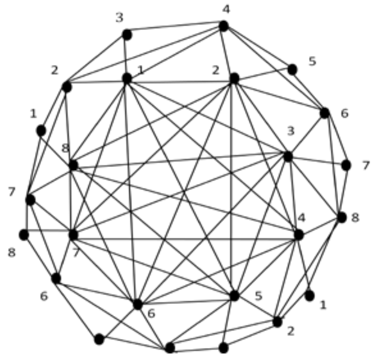

Figure: 3 $X_s [L (H_8)] =8$

5. STAR COLOURING OF CENTRAL GRAPH OF HELM GRAPH $C(H_n)$

5.1 Theorem: For Central graph of H_n, $X_s[C(H_n)] = 2n+1$.

Proof: Let $\{v_1,v_2,\ldots.v_n\}$ represent the pendant vertices of H_n, $\{v_{n+1},v_{n+2},.....v_{2n}\}$ represent the vertices of the cycle in H_n, v_{2n+1} represent the hub and v_{ij} be the introduced vertex to subdivide (v_i,v_j) where $i,j =1$ to $2n+1$.

In hub, the vertex subset $\{v_1, v_{2,\ldots.} v_n\}$ and the vertices v_{ij} for $i = n+1$ to $2n$, $j=2n+1$ form $k_{1,2n}$ star graph. Now assign the star colouring as follows. Assign the colours c_i to the pendant vertices v_i for $i=1$ to n, the colours c_i to the vertices of the cycle v_i for $i=n+1$ to $2n$. Also assign the colour c_{2n+1} to v_{2n+1}. Next assign the colours to v_{ij} of the cycle as c_{i+1} to v_{ij} for $i=n+1$ to $2n-1$, $j=2n+1$ and the colour c_{n+1} to v_{ij} for $i=2n$ and $j=2n+1$.

Every v_i, for $i=n+1$ to $2n$ has four adjacent vertices that is to be coloured with different colours. Since the colouring should be minimum, we try to repeat the colours that are already used. Assign the colours c_i, for $i=1$ to $n-1$ to v_{ij} for $i=n+1$ to $2n-1$, $j=i+1$ and the colour c_n to v_{ij} for $i=2n$, $j=n+1$. Also assign the colour c_{i+1} to v_{ij} for $i = 1$ to $n-1$, $j=i+n$ and colour c_1 to v_{ij} for $i=n$, $j=2n$.

This is clearly star colouring, since the four adjacent vertices v_{ij} to the vertex v_i get different colours and the colouring is also minimum.

Thus, $X_s[C(H_n)] = 2n+1$.

Example:

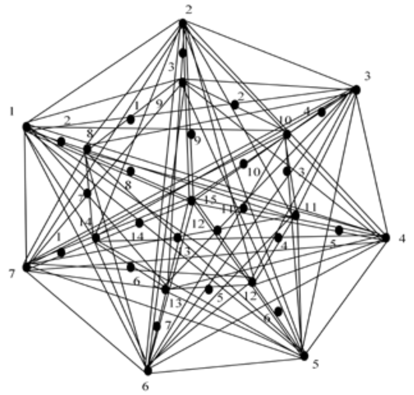

Figure: 4 $X_s[C(H_7)] =15$

6. STAR COLOURING OF LINE GRAPH OF WHEEL GRAPH $L(W_n)$

6.1 Theorem: For the Line Graph of W_n, $X_s[L(W_n)] = n-1$.

Proof: Let W_n be a wheel graph with n vertices. Let the vertex set of W_n be $\{v_1, v_2,\ldots\ldots\ldots\ldots v_n\}$ with v_n as the hub. Now $L(W_n)$ contains a complete graph $K_{n-1}$with $V(K_{n-1}) =\{x_1,x_2,\ldots\ldots\ldots x_{n-1}\}$ and a cycle C with $V(C) = \{u_1,u_2,\ldots\ldots\ldots u_{n-1}\}$. Now assign a star colouring as follows. Assign the colour c_i to the vertex v_i for $i=1, 2 \ldots n-1$.

Next we have to colour the vertices of the cycle. Suppose we want to introduce one more colour say c_n, we should colour any one of the vertex of the cycle C. In C, each vertex u_i is adjacent with u_{i+1},u_{i-1} for $i=2,3\ldots..n-1$ and x_i, x_{i+1} for $i=1,2,3\ldots..n-1$. u_1 is adjacent with u_{i+1},u_n ,x_i , x_{i+1} . u_n is adjacent with $u_{n-1},u_1,x_{n-1,}x_1$. But each u_i ε $V(C)$ has degree four. Therefore, the vertex which has been coloured as c_n cannot realizes its colour as c_n. Since in the neighbour set of the vertex, we cannot introduce new colours.

So assign only the existing colours to the vertex of the cycle C. Now assign c_i for $i=1, 2,\ldots\ldots..n-3$ to the vertex u_i and the colour c_{n-2} to the vertex u_1 and c_{n-1} to u_2. Thus further introduction of new colour is not possible. Hence colouring should be minimum and a star colouring. Thus $X_s[L(W_n)] = n-1$.

Example:

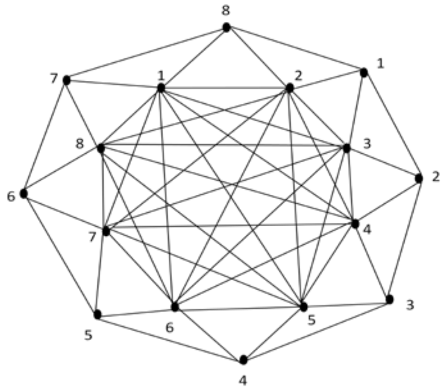

Figure: 5 $X_s[L(W_9)] = 8$.

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