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GENERALIZED DERIVATION ON LIE IDEAL IN PRIME RINGS

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ABSTRACT

Let R be a 2-torsion free prime ring R and U be non zero Lie ideal of prime ring R and F be a generalized derivation satisfied any condition

(i)F[x, y]=0 (ii) F(xoy)=0 , (iii) F(x) oF (y)=0 , (iv) [F(x), F(y)]=0 for all x, y $\in U$ *Then* $U \subseteq Z(R)$.

Key Words: *Prime Ring, Generalized Derivation, Lie ideal.*

1. INTRODUCTION

Let R be an associative ring and Z(R) denote the centre of ring R throughout the paper. For each x, y ∈ R, [x, y] means xy-yx and xoy means xy+yx. Recall that a ring R is prime if for any a, b ∈ R, aRb= {0} implies that a=0 or b=0. An additive mapping d:R→R is called a derivation if d(xy)=d(x)y+xd(y) for all x, y ϵ R. For a fixed a ∈ R, the mapping d_a : R→R defined by d_a (x)= [a, x] is called inner derivation.

In [2], Bresar coined the concept of generalized derivation in 2006. Accordingly, An additive mapping F: R→R is called a generalized derivation if F(xy)=F(x)y+xd(y) for all x, y ∈ R. It can be easily check that the concept of generalized derivation covered the concept of derivation as well as inner derivation.

Firstly, E.C. Posner shows the commutativity by the concept of derivation in prime ring, which states that the existence of a non zero centralizing derivation on a prime ring forces this ring to be commutative. Motivated by E.C. Posner, in [5], I.N. Herstein proved that if R is a ring with char(R)≠2 which has a nonzero derivation d such that [d(x),d(y)]=0 for all x, y ∈ R then R is commutative. In [9], H.E. Bell and M.N. Daif studied derivation d satisfying d([x, y])=0 for all x, y∈ R.

In[6,8], L. Oukhtite *et al.* generalized the some above result to σ-Lie ideal and σ -square closed Lie ideal in σ-prime rings.

A number of several authors in [11, 12, 1] extended the results in generalized derivation instead of derivation.

In this paper, we have established some results of [8, 10] in the case of generalized derivation instead of derivation.

2. PRELIMINARIES

Throughout the paper, we shall use some basic identities as follows:

Xo(yz)=(xoy)z – y[x, z]=y(xoz) + [x, y]z

(xy)oz=x(yoz) – [x, z]y=(xoz)y + x[y, z]

We shall use some known lemmas to prove our results:

Lemma 2.1: [1, 3.1] Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R such that $u^2 \in U$. If R admits a nonzero generalized derivation F with d such that [F(u),u]=0 for all u ∈ U and if d≠0 then $U \subseteq Z(R)$.

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Lemma 2.2: [7, 7] Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R such that [u, d(u)] ∈ Z(R) for all u ∈ U then U ⊆ $Z(R)$.

Lemma 2.3: [1, 2.6] Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R. If U is a commutative Lie ideal of R i.e. [u, v] =0 for all u, v ∈ U then U ⊆ $Z(R)$.

Lemma 2.4: [5, 5] Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R. If d is a nonzero derivation of R such that d(U)=0 then U ⊆ $Z(R)$.

3. MAIN RESULTS

Theorem 3.1: Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R .If F be a nonzero generalized derivation with nonzero derivation d such that F ([x, y]) =0 for all x, y ∈ U then U ⊆ $Z(R)$.

Proof: Let x, y ∈ U. Since F ([x, y]) =0 for all x, y ∈ U (1)

This implies F(xy)=F(yx) (2)

F([x, y]z)=F(z[x, y]) (3)

From F(xy)=F(yx), we get

[F(x), y]= [d(y), x] for all x, y ∈ U (4)

Replacing y by y^2 in (4)

[F(x), y] y=d(y) [y, x] (5)

Replacing x by xz in (5) – we obtain

d(y)z[y, x]=0 for all x, y, z ∈ U (6)

This implies d(y) U[y, x] =0 for all x, y ∈ U (7)

either d(y)=0 for all y ∈ U

this implies d(U)=0

Applying Lemma (2.3) we get

U ⊆ $Z(R)$

Or [y, x] = 0 for all x, y ∈ U

Then apply Lemma (2.2), we get

U ⊆ $Z(R)$.

Theorem 3.2: Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R. If F be a nonzero generalized derivation with nonzero derivation d such that [F(x), F(y)] =0 for all x, y ∈ U then U ⊆ $Z(R)$.

Proof: Let [F(x), F(y)] =0 for all x, y ∈ U (8)

Replacing x by xy in (8)

[F(xy), F(y)]=0 for all x, y ∈ U

Which gives us

F(x)[y, F(y)] + [x, F(y)]d(y) =0 for all x, y ∈ U (9)

If we replace x by yz in (9), then we get

[y, F(y)]zd(y) =0 for all x, y, z ∈ U (10)

Which implies

[y, F(y)] Ud(y) =0 for all y ∈ U (11)

Which implies either

[y, F(y)] =0 for all y ∈ U (12)

Then applying Lemma 2.1

U ⊆ $Z(R)$

Or

d(y) =0 for all y ∈ U.

which imlies

d(U) =0

Applying Lemma 2.4 we get

U ⊆ $Z(R)$.

Theorem 3.3: Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R. If F be a nonzero generalized derivation with nonzero derivation d such that F(xoy)=0 for all x, y ∈ U then U ⊆ $Z(R)$.

Proof: F(xoy) =0 for all x, y ∈ U (13)

Replacing x by xy in (12)—

F(xyoy) =0 w

Which implies

F((xoy) + x[y, y]) =0 for all x, y ∈ U (14)

Then we obtain

F((xoy)y) =0 for all x, y ∈ U (15)

This implies –

F(xoy)y + (xoy)d(y) =0 (16)

Using (12), we get

(xoy)d(y) =0 for all x,y ∈ U (17)

Replacing x by zx in (16)

(zxoy)d(y) =0 for all x, y, z ϵ U

(z(xoy) – [z, y]y)d(y) =0 for all x, y, z ∈ U (18)

Using (16) we get

[z, y]yd(y) =0 for all z, y ∈ U

This implies

[z, y] U d(y) =0 for all z, y ∈ U (19)

Then either

[z, y] =0 for all z, y ∈ U

Applying Lemma 2.3, we get

U ⊆ $Z(R)$

Or

d(y) =0 for all y ∈ U

then d(U) =0

Applying Lemma 2.2 we get

U ⊆ Z(R).

Theorem 3.4: Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R. If F be a nonzero generalized derivation with nonzero derivation d such that F(x) oF(y) =0 for all x, y ∈ U then U⊆ $Z(R)$.

Proof: Let F(x)oF(y) =0 for all x, y ∈ U (20)

Replacing y by xy in (19), we get

F(x)oF(xy) =0 for all x, y ∈ U

F(x)o(F(x)y + xd(y)) =0 for all x, y ∈ U (21)

F(x).F(x)y + F(x)xd(y) + F(x)yF(x) + xd(y)F(x) =0 for all x, y ∈ U (22)

We know—

F(x)oF(y) =0 implies F(x)F(y) = -F(y)F(x) (23)

And d(y)oF(x) =0 implies d(y)F(x) = -F(x)d(y) (24)

Using facts (22) and (23) in (21), we get—

-F(x)F(x)y + F(x)xd(y) + F(x)yF(x) - xF(x)d(y) =0 for all x, y ∈ U (25)

This implies

F(x)[y, F(x)] + [F(x), x]d(y) =0 for all x, y ∈ U (26)

Replacing y by y^2 in (25), we get

[F(x), x]yd(y) =0 for all x, y ∈ U

[F(x), x] U d(y) =0 for all x, y ∈ U

Then either

[F(x), x] =0 for all x ∈ U

Applying Lemma 2.1, we get

U ⊆ $Z(R)$

Or

$d(y) = 0$ for all $y \in U$

then $d(U) = 0$

Applying Lemma 2.4, we get—

$U \subseteq Z(R)$.

CONCLUSION

Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R. If d is nonzero derivation of R. If F be a nonzero generalized derivation with nonzero derivation d such that satisfying any one condition:

(i) $F[x, y]=0$ (ii) $[F(x),F(y)]=0$ (iii) $F(xoy)=0$ (iv) $F(x)oF(y)$ for all $x, y \in U$

Then $U \subseteq Z(R)$.

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