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SOME FIXED POINT THEOREMS IN TWO $\mathcal{M}$ - FUZZY METRIC SPACES

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ABSTRACT

In this paper we prove some fixed point theorems for generalized contraction mappings in two complete $\mathcal{M}$ - fuzzy metric spaces.

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Keywords and Phrases: *common fixed point and complete $\mathcal{M}$ - fuzzy metric space.*

1. INTRODUCTION

Fuzzy set was defined by Zadeh [10] in 1965, has lead to a rich growth of fuzzy mathematics. Kramosil and Michalek [6] introduced fuzzy metric space, George and Veeramani [4] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many authors Deng [2], Erceg [3] used the concept of fuzzy mathematics in different ways. Recently Sedghi and Shobe [7] introduced D* - metric space as a probable modification of the definition of D – metric introduced by Dhage, and prove some basic properties in D* - metric spaces. Using D*- metric concepts, Sedghi and Shobe define $\mathcal{M}$ -fuzzy metric space and proved a common fixed point theorem in it. In this paper we prove some fixed point theorems in two complete $\mathcal{M}$ - fuzzy metric spaces for contractive type mappings and non-expansive mappings by generalizing the results of Veerapandi et al [9] on fuzzy metric space.

Definition 1.1: A fuzzy set A in X is a function with domain X and values in [0, 1]

Definition: 1.2: A binary operation *: [0, 1] x [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions

i. * is associative and commutative,
ii. * is continuous,
iii. $a * 1 = a$ for all $a \in [0, 1]$,
iv. $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0, 1]$

Examples for continuous t-norm are $a * b = \min\{a, b\}$.

Definition: 1.3: [7] A 3-tuple (X, $\mathcal{M}$, *) is called $\mathcal{M}$ – fuzzy metric space if X is an arbitrary non-empty set, * is a continuous t – norm, and $\mathcal{M}$ is a fuzzy set on $X^3 \times (0,\infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$

(FM – 1) $\mathcal{M}(x, y, z, t) > 0$
(FM – 2) $\mathcal{M}(x, y, z, t) = 1$ iff $x = y = z$
(FM – 3) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function
(FM – 4) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \le \mathcal{M}(x, y, z, t + s)$
(FM – 5) $\mathcal{M}(x, y, z, .) : (0, \infty) \to [0, 1]$ is continuous
(FM – 6) $\lim_{t\to\infty} \mathcal{M}(x, y, z, t) = 1$

Lemma: 1.4: Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$- fuzzy metric space. Then for every $t > 0$ and for every $x, y \in X$. We have $\mathcal{M}(x, x, y, t) = \mathcal{M}(x, y, y, t)$.

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Lemma: 1.5: Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$- fuzzy metric space. Then $\mathcal{M}(x, y, z, t)$ is non-decreasing with respect to t, for all x, y, z in X.

Definition: 1.6: Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$-fuzzy metric space and $\{x_n\}$ be a sequence in X

a) $\{x_n\}$ is said to be converges to a point $x \in X$ if $\lim_{n\to\infty} \mathcal{M}(x, x, x_n, t) = 1$ for all $t > 0$

b) $\{x_n\}$is called Cauchy sequence if $\lim_{n\to\infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = 1$for all $t > 0$ and $p > 0$

c) A $\mathcal{M}$-fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Lemma: 1.7: Let $\{x_n\}$ be a sequence in a $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}$, *) with the condition (FM-6). If there exists a number $k \in (0, 1)$ such that $\mathcal{M}(x_n, x_{n+1}, x_{n+1}, kt) \geq \mathcal{M}(x_{n-1}, x_n, x_n, t)$ for all $t > 0$ and n =1, 2, 3 …, then $\{x_n\}$ is a Cauchy sequence.

Lemma 1.8: Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$ – fuzzy metric space with condition (FM-6). If there exists a number $k \in (0, 1)$ such that $\mathcal{M}(x, y, z, kt) \geq \mathcal{M}(x, y, z, t)$, for all $x, y, z \in X$ and $t > 0$, then $x = y = z$.

2. MAIN RESULTS

Theorem 2.1: Let (X, $\mathcal{M}_1$, *) and (Y, $\mathcal{M}_2$, *) be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying the following conditions.

$$\mathcal{M}_2(Tx, TSy, TSy, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t), \mathcal{M}_2(y, Tx, Tx, t) * \mathcal{M}_2(y, TSy, TSy, t)\} \quad (1)$$

$$\mathcal{M}_1(Sy, STx, STx, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t) * \mathcal{M}_1(x, STx, STx, t), \mathcal{M}_2(y, Tx, Tx, t)\} \quad (2)$$

for all x in X and y in Y where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and $\{y_n\}$ in Y, as follows:
$x_n = (ST)^n x_0$, $y_n = T(x_{n-1})$ for n = 1,2, . . . , We have

$$\begin{aligned}
\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) &= \mathcal{M}_1((ST)^n x_0, (ST)^{n+1}x_0, (ST)^{n+1}x_0, qt) \\
&= \mathcal{M}_1(ST(ST)^{n-1} x_0, ST(ST)^n x_0, ST(ST)^n x_0, qt) \\
&= \mathcal{M}_1(STx_{n-1}, STx_n, STx_n, qt) = \mathcal{M}_1(Sy_n, STx_n, STx_n, qt) \\
&\geq \min\{\mathcal{M}_1(x_n, Sy_n, Sy_n, t) * \mathcal{M}_1(x_n, STx_n, STx_n, t), \mathcal{M}_2(y_n, Tx_n, Tx_n, t)\} \\
&= \min\{\mathcal{M}_1(x_n, x_n, x_n, t) * \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\} \\
&= \min\{\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\} \\
&\geq \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t).
\end{aligned}$$

$$\begin{aligned}
\text{Now, } \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t) &= \mathcal{M}_2(Tx_{n-1}, Tx_n, Tx_n, t) = \mathcal{M}_2(Tx_{n-1}, TSy_n, TSy_n, t) \\
&\geq \min\{\mathcal{M}_1(x_{n-1}, Sy_n, Sy_n, t/q), \mathcal{M}_2(y_n, Tx_{n-1}, Tx_{n-1}, t/q) * \mathcal{M}_2(y_n, TSy_n, TSy_n, t/q)\} \text{ (Since by (1))} \\
&= \min\{\mathcal{M}_1(x_{n-1}, x_n, x_n, t/q), \mathcal{M}_2(y_n, y_n, y_n, t/q) * \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t/q)\} \\
&= \min\{\mathcal{M}_1(x_{n-1}, x_n, x_n, t/q), \mathcal{M}(y_n, y_{n+1}, y_{n+1}, t/q)\} \\
&\geq \mathcal{M}_1(x_{n-1}, x_n, x_n, t/q)
\end{aligned}$$

$$\begin{aligned}
\text{Hence, } \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) &\geq \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}_1(x_{n-1}, x_n, x_n, t/q) \quad . \quad . \quad . \\
&\geq \mathcal{M}_1(x_0, x_1, x_1, t/q^{2n-1}) \to 1 \text{ as } n \to \infty \quad (\text{Since } q < 1).
\end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in X, Since (X, $\mathcal{M}_1$, *) is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in Y and it converges to a point w in Y. Now we prove Tz = w. Suppose $Tz \neq w$. We have,

$$\begin{aligned}
\mathcal{M}_2(Tz, w, w, qt) &= \lim_{n\to\infty} \mathcal{M}_2(Tz, y_{n+1}, y_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_2(Tz, TSy_n, TSy_n, qt) \\
&\geq \lim_{n\to\infty} \min \mathcal{M}_1(z, Sy_n, Sy_n, t), \mathcal{M}_2(y_n, Tz, Tz, t) * \mathcal{M}_2(y_n, TSy_n, TSy_n, t)\} \\
&= \lim_{n\to\infty} \min \mathcal{M}_1(z, x_n, x_n, t), \mathcal{M}_2(y_n, Tz, Tz, t) * \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\} \\
&= \min\{\mathcal{M}_1(z, z, z, t), \mathcal{M}_2(w, Tz, Tz, t) * \mathcal{M}_2(w, w, w, t)\} \\
&= \min\{1, \mathcal{M}_2(w, Tz, Tz, t) * 1\} \geq \mathcal{M}_2(w, Tz, Tz, t) \\
&\geq \mathcal{M}_2(Tz, w, w, t) \text{ (Since } q < 1\text{), which is a contradiction.}
\end{aligned}$$

Thus Tz = w. Now, we prove Sw = z. Suppose Sw ≠ z, we have

$$\begin{aligned}\mathcal{M}_1(Sw, z, z, qt) &= \lim_{n\to\infty} \mathcal{M}_1(Sw, x_{n+1}, x_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_1(Sw, STx_n, STx_n, qt)\\ &\geq \lim_{n\to\infty} \min \mathcal{M}_1(x_n, Sw, Sw, t)*\mathcal{M}_1(x_n, STx_n, STx_n, t), \mathcal{M}_2(w, Tx_n, Tx_n, t)\}\\ &\geq \lim_{n\to\infty} \min \mathcal{M}_1(x_n, Sw, Sw, t)*\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(w, y_{n+1}, y_{n+1}, t)\}\\ &= \min\{\mathcal{M}_1(z, Sw, Sw, t)*1, 1\} \geq \mathcal{M}_1(z, Sw, Sw, t) \text{ (Since } q > 1),\end{aligned}$$

which is a contradiction. Thus Sw = z. Therefore we have STz = Sw = z and TSw = Tz = w. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: Let z′ be another fixed point of ST such that z = z′. We have

$$\begin{aligned}\mathcal{M}_1(z, z', z', qt) &= \mathcal{M}_1(STz, STz', STz', qt)\\ &\geq \min\{\mathcal{M}_1(z', STz, STz, t)*\mathcal{M}_1(z', STz', STz', t), \mathcal{M}_2(Tz, Tz', Tz', t)\}\\ &= \min\{\mathcal{M}_1(z', z, z, t), \mathcal{M}_2(Tz, Tz', Tz', t)\} \geq \mathcal{M}_2(Tz, Tz', Tz', t)\end{aligned}$$

Also we have,
$$\begin{aligned}\mathcal{M}_2(Tz, Tz', Tz', t) &= \mathcal{M}_2(Tz, TSTz', TSTz', t)\\ &\geq \min\{\mathcal{M}_1(z, STz', STz', t/q), \mathcal{M}_2(Tz', Tz, Tz, t/q)*\mathcal{M}_2(Tz', TSTz', TSTz', t/q)\}\\ &\geq \min\{\mathcal{M}_1(z, z', z', t/q), \mathcal{M}_2(Tz', Tz, Tz, t/q)\}\\ &\geq \mathcal{M}_1(z, z', z', t/q)\end{aligned}$$

Hence, $\mathcal{M}_1(z, z', z', t/q) \geq \mathcal{M}_2(Tz, Tz', Tz', t) \geq \mathcal{M}_1(z, z', z', t/q)$

which is a contradiction. Thus z = z′. So the point z is the unique fixed point of ST.

Similarly, we prove the point w is also a unique fixed point of TS.

Corollary 2.2: Let (X, $\mathcal{M}$, *) be a complete $\mathcal{M}$- fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions.

$$\mathcal{M}(Tx, TSy, TSy, qt) \geq \min\{\mathcal{M}(x, Sy, Sy, t), \mathcal{M}(y, Tx, Tx, t)*\mathcal{M}(y, TSy, TSy, t)\}$$

$$\mathcal{M}(Sy, STx, STx, qt) \geq \min\{\mathcal{M}(x, Sy, Sy, t)*\mathcal{M}(x, STx, STx, t), \mathcal{M}(y, Tx, Tx, t)\}$$

for all x, y in X where q < 1, then ST has a unique fixed point z in X and TS has a unique fixed point w in X. Further Tz = w and Sw = z.

Theorem 2.3: Let (X, $\mathcal{M}_1$,*) and (Y, $\mathcal{M}_2$,*) be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying following conditions.

$$\mathcal{M}_2(Tx, TSy, TSy, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t), \mathcal{M}_2(y, Tx, Tx, t), \mathcal{M}_2(y, Tx, Tx, t)*\mathcal{M}_2(y, TSy, TSy, t)\} \quad (1)$$

$$\mathcal{M}_1(Sy, STx, STx, qt) \geq \min\{\mathcal{M}_2(y, Tx, Tx, t), \mathcal{M}_1(x, Sy, Sy, t), \mathcal{M}_1(x, Sy, Sy, t)*\mathcal{M}_1(x, STx, STx, t)\} \quad (2)$$

for all x in X and y in Y where q < 1, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and $\{y_n\}$ in Y, as follows
$x_n = (ST)^n x_0$, $y_n = T(x_{n-1})$ for n =1, 2 . . . , We have

$$\begin{aligned}\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) &= \mathcal{M}_1((ST)^n x_0, (ST)^{n+1}x_0, (ST)^{n+1}x_0, qt)\\ &= \mathcal{M}_1(ST(ST)^{n-1}x_0, ST(ST)^n x_0, ST(ST)^n x_0, qt)\\ &= \mathcal{M}_1(STx_{n-1}, STx_n, STx_n, qt) = \mathcal{M}_1(Sy_n, STx_n, STx_n, qt)\\ &\geq \min\{\mathcal{M}_1(y_n, Tx_n, Tx_n, t), \mathcal{M}_1(x_n, Sy_n, Sy_n, t), \mathcal{M}_1(x_n, Sy_n, Sy_n, t) * \mathcal{M}_1(x_n, STx_n, STx_n, t)\}\\ &= \min\{\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(x_n, x_n, x_n, t), \mathcal{M}_1(x_n, x_n, x_n, t) * \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t)\}\\ &= \min\{\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), 1, 1*\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t)\}\\ &\geq \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t).\end{aligned}$$

Now,
$$\begin{aligned}\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, qt) &= \mathcal{M}_2(Tx_{n-1}, Tx_n, Tx_n, qt) = \mathcal{M}_2(Tx_{n-1}, TSy_n, TSy_n, qt)\\ &\geq \min\{\mathcal{M}_1(x_{n-1}, Sy_n, Sy_n, t), \mathcal{M}_2(y_n, Tx_{n-1}, Tx_{n-1}, t),\\ &\qquad \mathcal{M}_2(y_n, Tx_{n-1}, Tx_{n-1}, t)*\mathcal{M}_2(y_n, TSy_n, TSy_n, t)\}\end{aligned}$$

$= \min \{\mathcal{M}_1 (x_{n-1}, x_n, x_n, t), \mathcal{M}_2 (y_n, y_n, y_n, t), \mathcal{M}_2 (y_n, y_n, y_n, t)*\mathcal{M}_2 (y_n, y_{n+1}, y_{n+1}, t)\}$
$= \min \{\mathcal{M}_1 (x_{n-1}, x_n, x_n, t), 1, 1*\mathcal{M}_2 (y_n, y_{n+1}, y_{n+1}, t)\}$
$= \mathcal{M}_1 (x_{n-1}, x_n, x_n, t)$

$\mathcal{M}_1 (x_n, x_{n+1}, x_{n+1}, t) \geq \mathcal{M}_2 (y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}_1 (x_{n-1}, x_n, x_n, t/q)\ldots$
$\geq \mathcal{M}_1 (x_0, x_1, x_1, t/q^{2n-1}) \rightarrow 1$ as $n\rightarrow\infty$. (Since $q < 1$)

Thus $\{x_n\}$ is a Cauchy sequence in X. Since (X, $\mathcal{M}_1$, *) is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in Y and it converges to a point w in Y. Now we prove Tz = w. Suppose Tz ≠ w. We have,

$$\mathcal{M}_2 (Tz, w, w, qt) = \lim_{n\to\infty} \mathcal{M}_2(Tz, y_{n+1}, y_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_2 (Tz, TSy_n, TSy_n, qt)$$
$$\geq \lim_{n\to\infty} \min \mathcal{M}_1 (z, Sy_n, Sy_n, t), \mathcal{M}_2(y_n, Tz, Tz, t)\ \mathcal{M}_2 (y_n, Tz, Tz, t) * \mathcal{M}_2 (y_n, TSy_n, TSy_n, t)\}$$
$$= \lim_{n\to\infty} \min \mathcal{M}_1(z, x_n, x_n, t), \mathcal{M}_2(y_n, Tz, Tz, t), \mathcal{M}_2 (y_n, Tz, Tz, t) * \mathcal{M}_2 (y_n, y_{n+1}, y_{n+1}, t)\}$$
$$= \min \{\mathcal{M}_1 (z, z, z, t), \mathcal{M}_2 (w, Tz, Tz, t), \mathcal{M}_2 (w, Tz, Tz, t) * \mathcal{M}_2 (w, w, w, t)\}$$
$$= \min \{1, \mathcal{M}_2 (w, Tz, Tz, t), \mathcal{M}_2 (w, Tz, Tz, t) * 1\}$$
$$\geq \mathcal{M}_2 (w, Tz, Tz, t)$$ (Since $q < 1$). Which is a contradiction.

Thus Tz = w. Now we prove Sw = z. Suppose Sw ≠ z. We have

$$\mathcal{M}_1 (Sw, z, z\ qt) = \lim_{n\to\infty} \mathcal{M}_1(Sw, x_{n+1}, x_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_1(Sw, STx_n, STx_n, qt)$$
$$\geq \lim_{n\to\infty} \min \{ \mathcal{M}_2(w, Tx_n, Tx_n, t), \mathcal{M}_1(x_n, Sw, Sw, t)\mathcal{M}_1 (x_n, Sw, Sw, t) * \mathcal{M}_1 (x_n, STx_n, STx_n, t)\}$$
$$= \lim_{n\to\infty} \min \{ \mathcal{M}_2(w, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(x_n, Sw, Sw, t)\mathcal{M}_1 (x_n, Sw, Sw, t) * \mathcal{M}_1 (x_n, x_{n+1}, x_{n+1}, t)\}$$
$$= \min \{\mathcal{M}_2 (w, w, w, t), \mathcal{M}_1 (z, Sw, Sw, t), \mathcal{M}_1 (z, Sw, Sw, t) * \mathcal{M}_1 (z, z, z, t)\}$$
$$\geq \mathcal{M}_1 (z, Sw, Sw, t)$$ (Since $q < 1$). Which is a contradiction.

Thus Sw = z. Therefore we have STz = Sw = z and TSw = Tz = w.

Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: Let z′ be another fixed point of ST such that z = z′. We have

$$\mathcal{M}_1 (z', z, z, qt) = \mathcal{M}_1 (STz', STz, STz, qt)$$
$$\geq \min \{\mathcal{M}_2 (Tz', Tz, Tz, t), \mathcal{M}_1 (z, STz', STz', t), \mathcal{M}_1 (z, STz', STz', t) * \mathcal{M}_1 (z, STz, STz, t)\}$$
$$\geq \min \{\mathcal{M}_2 (Tz', Tz, Tz, t), \mathcal{M}_1 (z, z', z', t), \mathcal{M}_1 (z, z', z', t) * M_1 (z, z, z, t)\}$$
$$\geq \mathcal{M}_2 (Tz', Tz, Tz, t).$$

Also we have, $\mathcal{M}_2 (Tz', Tz, Tz, qt) = \mathcal{M}_2 (Tz', TSTz, TSTz, qt)$
$\geq \min \{\mathcal{M}_1 (z', STz, STz, t), \mathcal{M}_2 (Tz, Tz', Tz', t),$
$\mathcal{M}_2 (Tz, Tz', Tz', t) * \mathcal{M}_2 (Tz, TSTz, TSTz, t)\}$
$\geq \min \{\mathcal{M}_1 (z', z, z, t), \mathcal{M}_2 (Tz, Tz', Tz', t), \mathcal{M}_2 (Tz, Tz', Tz', t) * \mathcal{M}_2 (Tz, Tz, Tz, t)\}$
$\geq \mathcal{M}_1 (z', z, z, t)$

$$\mathcal{M}_2 (Tz', Tz, Tz, t) \geq \mathcal{M}_1 (z', z, z, t/q)$$

Hence, $\mathcal{M}_1 (z', z, z, qt) \geq \mathcal{M}_2 (Tz', Tz, Tz, t) \geq \mathcal{M}_1 (z, z', z', t/q)$ which is a contradiction.

Thus z = z′. So the point z is the unique fixed point of ST.

Similarly, we prove the point w is also a unique fixed point of TS.

Corollary 2.4: Let (X, $\mathcal{M}$, *) be a complete $\mathcal{M}$- fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions.
$\mathcal{M}$ (Tx, TSy, TSy, qt) ≥ min {$\mathcal{M}$ (x, Sy, Sy, t), $\mathcal{M}$ (y, Tx, Tx, t), $\mathcal{M}$ (y, Tx, Tx, t)*$\mathcal{M}$ (y, TSy, TSy, t)}

$\mathcal{M}$ (Sy, STx, STx, qt) ≥ min {$\mathcal{M}$ (y, Tx, Tx, t), $\mathcal{M}$ (x, Sy, Sy, t), $\mathcal{M}$ (x, Sy, Sy, t)*$\mathcal{M}$ (x, Tx, Tx, t)}

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X. Further Tz = w and Sw = z.

Theorem 2.5: Let (X, $\mathcal{M}_1$, *) and (Y, $\mathcal{M}_2$, *) be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying following conditions.

$$\mathcal{M}_2(Tx, TSy, TSy, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t), \mathcal{M}_2(y, Tx, Tx, t), \mathcal{M}_2(y, TSy, TSy, t), \mathcal{M}_1(x, STx, STx, t), \mathcal{M}_1(Sy, STx, STx, t)\} \quad (1)$$

$$\mathcal{M}_1(Sy, STx, STx, qt) \geq \min\{\mathcal{M}_2(y, Tx, Tx, t), \mathcal{M}_1(x, Sy, Sy, t), \mathcal{M}_1(x, STx, STx, t), \mathcal{M}_2(Tx, TSy, TSy, t), \mathcal{M}_2(y, TSy, TSy, t)\} \quad (2)$$

for all x in X and y in Y where q < 1, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and $\{y_n\}$ in Y, as follows,
$x_n = (ST)^n x_0$, $y_n = T(x_{n-1})$ for n = 1,2, . . . , We have

$$\begin{aligned}\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) &= \mathcal{M}_1((ST)^n x_0, (ST)^{n+1} x_0, (ST)^{n+1} x_0, qt)\\ &= \mathcal{M}_1(ST(ST)^{n-1} x_0, ST(ST)^n x_0, ST(ST)^n x_0, qt)\\ &= \mathcal{M}_1(ST(x_{n-1}), STx_n, STx_n, qt) = \mathcal{M}_1(Sy_n, STx_n, STx_n, qt)\\ &\geq \min\{\mathcal{M}_2(y_n, Tx_n, Tx_n, t), \mathcal{M}_1(x_n, Sy_n, Sy_n, t), \mathcal{M}_1(x_n, STx_n, STx_n, t),\\ &\qquad \mathcal{M}_2(Tx_n, TSy_n, TSy_n, t), \mathcal{M}_2(y_n, TSy_n, TSy_n, t)\}\\ &= \min\{\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(x_n, x_n, x_n, t),\\ &\qquad \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(y_{n+1}, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\}\\ &= \min\{\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), 1, \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), 1, \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\}\\ &\geq \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t).\end{aligned}$$

Also, we have
$$\begin{aligned}\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, qt) &= \mathcal{M}_2(Tx_{n-1}, Tx_n, Tx_n, qt) = \mathcal{M}_2(Tx_{n-1}, TSy_n, TSy_n, qt)\\ &\geq \min\{\mathcal{M}_1(x_{n-1}, Sy_n, Sy_n, t), \mathcal{M}_2(y_n, Tx_{n-1}, Tx_{n-1}, t), \mathcal{M}_2(y_n, TSy_n, TSy_n, t),\\ &\qquad \mathcal{M}_1(x_{n-1}, STx_{n-1}, STx_{n-1}, t), \mathcal{M}_1(Sy_n, STx_{n-1}, STx_{n-1}, t)\}\\ &= \min\{\mathcal{M}_1(x_{n-1}, x_n, x_n, t), \mathcal{M}_2(y_n, y_n, y_n, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t),\\ &\qquad \mathcal{M}_1(x_{n-1}, x_n, x_n, t), \mathcal{M}_1(x_{n-1}, x_n, x_n, t)\}\\ &= \min\{\mathcal{M}_1(x_{n-1}, x_n, x_n, t), 1, \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(x_{n-1}, x_n, x_n, t), \mathcal{M}_1(x_{n-1}, x_n, x_n, t)\}\\ &\geq \mathcal{M}_1(x_{n-1}, x_n, x_n, t)\end{aligned}$$

Now,
$$\begin{aligned}\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) &\geq \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}_1(x_{n-1}, x_n, x_n, t/q) \quad . \quad . \quad .\\ &\geq \mathcal{M}_1(x_0, x_1, x_1, t/q^{2n-1}) \quad \to 1 \text{ as } n \to \infty \text{ (Since } q < 1)\end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in X. Since (X, $\mathcal{M}_1$, *) is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in Y and it converges to a point w in Y. Now, we prove Tz = w, we have

$$\begin{aligned}\mathcal{M}_2(Tz, w, w, qt) &= \lim_{n\to\infty} \mathcal{M}_2(Tz, y_{n+1}, y_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_2(Tz, TSy_n, TSy_n, qt)\\ &\geq \lim_{n\to\infty} \min\{\mathcal{M}_1(z, Sy_n, Sy_n, t), \mathcal{M}_2(y_n, Tz, Tz, t),\\ &\qquad \mathcal{M}_2(y_n, TSy_n, TSy_n, t)\ \mathcal{M}_1(z, STz, STz, t), \mathcal{M}_1(Sy_n, STz, STz, t)\}\\ &= \lim_{n\to\infty} \min\{\mathcal{M}_1(z, x_n, x_n, t), \mathcal{M}_2(y_n, Tz, Tz, t),\\ &\qquad \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\ \mathcal{M}_1(z, STz, STz, t), \mathcal{M}_1(x_n, STz, STz, t)\}\\ &= \min\{1, \mathcal{M}_2(w, Tz, Tz, t), 1, \mathcal{M}_1(z, STz, STz, t), \mathcal{M}_1(z, STz, STz, t)\}\\ &\geq \mathcal{M}_1(z, STz, STz, t)\end{aligned}$$

$$\begin{aligned}\mathcal{M}_1(z, STz, STz, qt) &= \lim_{n\to\infty} \mathcal{M}_1(x_n, STz, STz, qt) = \lim_{n\to\infty} \mathcal{M}_1(Sy_n, STz, STz, qt)\\ &\geq \lim_{n\to\infty} \min\{\mathcal{M}_2(y_n, Tz, Tz, t), \mathcal{M}_1((z, Sy_n, Sy_n, t),\\ &\qquad \mathcal{M}_1(z, STz, STz, t), \mathcal{M}_2(Tz, TSy_n, TSy_n, t), \mathcal{M}_2(y_n, TSy_n, TSy_n, t)\}\\ &= \lim_{n\to\infty} \min\{\mathcal{M}_2(y_n, Tz, Tz, t), \mathcal{M}_1((z, x_n, x_n, t),\\ &\qquad \mathcal{M}_1(z, STz, STz, t), \mathcal{M}_2(Tz, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\}\\ &= \min\{\mathcal{M}_2(w, Tz, Tz, t), 1, \mathcal{M}_1(z, STz, STz, t), \mathcal{M}_2(Tz, w, w, t), 1\}\\ &\geq \mathcal{M}_2(Tz, w, w, t)\end{aligned}$$

Hence, $\mathcal{M}_2(Tz, w, w, qt) \geq \mathcal{M}_2(Tz, w, w, t/q)$

Thus Tz = w. Now, we prove Sw = z. Suppose Sw ≠ z.

$$\begin{aligned}\mathcal{M}_1(Sw, z, z, qt) &= \lim_{n\to\infty} \mathcal{M}_1(Sw, x_{n+1}, x_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_1(Sw, STx_n, STx_n, qt)\\
&\geq \lim_{n\to\infty} \min\{\mathcal{M}_2(w, Tx_n, Tx_n, t), \mathcal{M}_1(x_n, Sw, Sw, t)\\
&\qquad \mathcal{M}_1(x_n, STx_n, STx_n, t), \mathcal{M}_2(Tx_n, TSw, Tsw, t), \mathcal{M}_2(y_n, TSx_n, TSx_n, t)\}\\
&= \lim_{n\to\infty} \min\{\mathcal{M}_2(w, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(x_n, Sw, Sw, t),\\
&\qquad \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_1(y_{n+1}, TSw, TSw, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)\}\\
&= \min\{1, \mathcal{M}_1(z, Sw, Sw, t), 1, \mathcal{M}_1(w, TSw, TSw, t), 1\}\\
&\geq \mathcal{M}_2(w, TSw, TSw, t)\end{aligned}$$

$$\begin{aligned}\text{Now, } \mathcal{M}_2(w, TSw, TSw, qt) &= \lim_{n\to\infty} \mathcal{M}_2(y_{n+1}, TSw, TSw, qt) = \lim_{n\to\infty} \mathcal{M}_2(Tx_n, TSw, TSw, qt)\\
&\geq \lim_{n\to\infty} \min\{\mathcal{M}_1(x_n, Sw, Sw, t), \mathcal{M}_1(w, Tx_n, Tx_n, t),\\
&\qquad \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(x_n, STx_n, STx_n, t), \mathcal{M}_1(Sw, STx_n, STx_n, t)\}\\
&= \lim_{n\to\infty} \min\{\mathcal{M}_2(x_n, Sw, Sw, t), \mathcal{M}_1(w, y_{n+1}, y_{n+1}, t)\\
&\qquad \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_1(Sw, x_{n+1}, x_{n+1}, t)\}\\
&= \min\{\mathcal{M}_1(z, Sw, Sw, t), 1, \mathcal{M}_2(w, TSw, TSw, t), 1, \mathcal{M}_1(Sw, z, z, t)\}\end{aligned}$$

$\geq \mathcal{M}_1(Sw, z, z, t)$

Hence, $\mathcal{M}_1(Sw, z, z, qt) \geq \mathcal{M}_2(w, TSw, TSw, t) \geq \mathcal{M}_1(Sw, z, z, t/q)$. Which is a contradiction.

Thus Sw = z. Therefore we have STz = Sw = z and TSw = Tz = w.

Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: Let z′ be the another fixed point of ST such that z ≠ z′.

$$\begin{aligned}\text{Now, } \mathcal{M}_1(z, z', z', qt) &= \mathcal{M}_1(Sw, STz', STz', qt)\\
&\geq \min\{\mathcal{M}_2(w, Tz', Tz', t), \mathcal{M}_1(z', Sw, Sw, t), \mathcal{M}_1(z', STz', STz', t),\\
&\qquad \mathcal{M}_2(Tz', TSw, TSw, t), \mathcal{M}_2(w, TSw, TSw, t)\}\\
&= \min\{\mathcal{M}_2(w, Tz', Tz', t), \mathcal{M}_1(z', z, z, t), 1, \mathcal{M}_2(Tz', w, w, t), 1\}\\
&\geq \mathcal{M}_2(Tz', w, w, t)\end{aligned}$$

$$\begin{aligned}\mathcal{M}_2(Tz', w, w, qt) &= \mathcal{M}_2(Tz', TSw, TSw, t)\\
&\geq \min\{\mathcal{M}_1(z', Sw, Sw, t), \mathcal{M}_2(w, Tz', Tz', t), \mathcal{M}_1(z', TSz', TSz', t),\\
&\qquad \mathcal{M}_1(z', z, z, t), \mathcal{M}_1(z, STz', STz', t)\}\\
&= \min\{\mathcal{M}_1(z', z, z, t), \mathcal{M}_2(w, Tz', Tz', t), 1, \mathcal{M}_1(z', z, z, t), \mathcal{M}_1(z, z', z', t)\}\\
&\geq \mathcal{M}_1(z, z', z', t)\end{aligned}$$

Hence, $\mathcal{M}_1(z, z', z', qt) \geq \mathcal{M}_2(Tz', w, w, t) \geq \mathcal{M}_1(z, z', z', t/q)$, which is a contradiction.

Thus z = z′. So the point z is the unique fixed point of ST.

Similarly, we prove the point w is also a unique fixed point of TS.

Corollary 2.6: Let (X, $\mathcal{M}$, *) be a complete $\mathcal{M}$-fuzzy metric space. If S and Tare mappings from X into itself satisfying the following conditions.

$$\begin{aligned}\mathcal{M}(Tx, TSy, TSy, qt) &\geq \min\{\mathcal{M}(x, Sy, Sy, t), \mathcal{M}(y, Tx, Tx, t), \mathcal{M}(y, TSy, TSy, t),\\
&\qquad \mathcal{M}(x, STx, STx, t), \mathcal{M}(Sy, STx, STx, t)\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}(Sy, STx, STx, qt) &\geq \min\{\mathcal{M}(y, Tx, Tx, t), \mathcal{M}(x, Sy, Sy, t), \mathcal{M}(x, STx, STx, t),\\
&\qquad \mathcal{M}(Tx, TSy, TSy, t), \mathcal{M}(y, TSy, TSy, t)\}\end{aligned}$$

for all x, y in X where q < 1, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Theorem 2.7: Let (X, $\mathcal{M}_1$, *) and (Y, $\mathcal{M}_2$,*) be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying the following conditions.

$$\mathcal{M}_2 (Tx, TSy, TSy, t) \geq \min\{\mathcal{M}_1 (x ,Sy, Sy, t), \mathcal{M}_1 (Sy, STx, STx, t), \mathcal{M}_2 (y, Tx, Tx, t) * \mathcal{M}_2 (y, TSy, TSy, t), \mathcal{M}_1 (x, STx, STx, t)\} \quad (1)$$

$$\mathcal{M}_1 (Sy, STx, STx, t) \geq \min\{\mathcal{M}_1 (x, Sy, Sy, t) * \mathcal{M}_1 (x, STx, STx, t), \mathcal{M}_2 (y, TSy, TSy, t), \mathcal{M}_2 (y, Tx, Tx, t), \mathcal{M}_2 (Tx, TSy, TSy, t)\} \quad (2)$$

for all x in X and y in Y where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further $Tz = w$ and $Sw = z$.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and $\{y_n\}$ in Y, as follows. $x_n = (ST)^n x_0$, $y_n = T(x_n\text{-}1)$ for n = 1, 2 . . . , we have

$$\begin{aligned}
\mathcal{M}_1 (x_n, x_{n+1}, x_{n+1}, qt) &= \mathcal{M}_1((ST)^n x_0, (ST)^{n+1}x_0, (ST)^{n+1}x_0, qt) \\
&= \mathcal{M}_1 (ST (ST)^{n-1}x_0, ST(ST)^n x_0, ST(ST)^n x_0, qt) \\
&= \mathcal{M}_1 (STx_{n-1}, STx_n, STx_n, qt) = \mathcal{M}_1 (Sy_n, STx_n, STx_n, qt) \\
&\geq \min \{\mathcal{M}_1 (x_n, Sy_n, Sy_n, t)*\mathcal{M}_1 (x_n, STx_n, STx_n, t), \\
&\qquad \mathcal{M}_2(y_n, TSy_n, TSy_n , t), \mathcal{M}_2(y_n, Tx_n, Tx_n, t), \mathcal{M}_2(Tx_n, TSy_n, TSy_n, t)\} \\
&= \min \{\mathcal{M}_1(x_n, x_n, x_n, t)*\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \\
&\qquad \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_{n+1}, y_{n+1}, y_{n+1}, t)\} \\
&= \min\{1*\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), 1\} \\
&\geq \mathcal{M}_2 (y_n, y_{n+1}, y_{n+1}, t)
\end{aligned}$$

$$\begin{aligned}
\text{Now, } \mathcal{M}_2 (y_n, y_{n+1}, y_{n+1}, qt) &= \mathcal{M}_2 (Tx_{n-1}, Tx_n, Tx_n, qt) = \mathcal{M}_2 (Tx_{n-1}, TSy_n, TSy_n, qt) \\
&\geq \min \{\mathcal{M}_1 (x_{n-1}, Sy_n, Sy_n, t), \mathcal{M}_1 (Sy_n, STx_{n-1}, ST x_{n-1}, t), \\
&\qquad \mathcal{M}_2(y_n, Tx_{n-1}, Tx_{n-1}, t)*\mathcal{M}_2(y_n, TSy_n, TSy_n, t), \mathcal{M}_1(x_{n-1}, STx_{n-1}, STx_{n-1} , t)\} \\
&= \min\{\mathcal{M}_1(x_{n-1}, x_n, x_n, t), 1, 1*\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(x_{n-1}, x_n, x_n, t)\} \\
&\geq \mathcal{M}_1 (x_{n-1}, x_n, x_n, t).
\end{aligned}$$

$$\begin{aligned}
\text{Hence, } \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) &\geq \mathcal{M}_2 (y_n, y_{n+1}, y_{n+1},t) \geq \mathcal{M}_1 (x_{n-1}, x_n, x_n, t/q) \; . \; . \; . \\
&\geq \mathcal{M}_1 (x_0, x_1, x_1, t/q^{2n-1}) \rightarrow 1 \text{ as } n\rightarrow\infty . \text{ (Since } q < 1)
\end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in $(X, \mathcal{M}_1,*)$. Since $(X, \mathcal{M}_1,*)$ is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in $(Y, \mathcal{M}_2, *)$. Since $(Y, \mathcal{M}_2, *)$ is complete, it converges to a point w in Y.

Now we prove $Tz = w$. Suppose $Tz \neq w$. We have,

$$\begin{aligned}
\mathcal{M}_2(Tz, w, w, qt) &= \lim_{n\to\infty} \mathcal{M}_2 (Tz, y_{n+1}, y_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_2 (Tz, TSy_n, TSy_n, qt) \\
&\geq \lim_{n\to\infty} \min\{\mathcal{M}_1 (z, Sy_n, Sy_n, t), \mathcal{M}_1(Sy_n, STz, STz, t) \\
&\qquad \mathcal{M}_2(y_n, Tz, Tz, t)*\mathcal{M}_2(y_n, TSy_n, TSy_n, t), \mathcal{M}_1(z, STz, STz, t)\} \\
&= \lim_{n\to\infty} \min \{\mathcal{M}_1 (z, x_n, x_n, t), \mathcal{M}_1 (x_n, STz, STz, t) \\
&\qquad \mathcal{M}_2(y_n, Tz, Tz, t)*\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(z, STz, STz, t)\} \\
&\geq \mathcal{M}_1 (z, STz, STz, t).
\end{aligned}$$

$$\begin{aligned}
\text{Now, } \mathcal{M}_1 (z, STz, STz, qt) &= \lim_{n\to\infty} \mathcal{M}_1 (x_n, STz, STz, qt) = \lim_{n\to\infty} \mathcal{M}_1 (Sy_n, STz, STz, qt) \\
&\geq \lim_{n\to\infty} \min\{\mathcal{M}_1 (z, Sy_n, Sy_n, t) * \mathcal{M}_1 (z, STz, STz, t) \\
&\qquad \mathcal{M}_2(y_n, TSy_n, TSy_n, t), \mathcal{M}_2(y_n, Tz, Tz, t), \mathcal{M}_2(Tz, TSy_n, TSy_n, t)\} \\
&= \lim_{n\to\infty} \min\{\mathcal{M}_1 (z, x_n, x_n, t) * \mathcal{M}_1 (z, STz, STz, t), \\
&\qquad \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_n, Tz, Tz, t), \mathcal{M}_2(Tz, y_{n+1}, y_{n+1}, t)\} \\
&= \min \{1*\mathcal{M}_1(z, STz, STz , t), 1, \mathcal{M}_2(w, Tz, Tz, t), \mathcal{M}_2(Tz, w, w, t)\} \\
&\geq \mathcal{M}_2 (Tz, w, w, t).
\end{aligned}$$

Hence , $\mathcal{M}_2(Tz, w, w, qt) \geq \mathcal{M}_1(z, STz, STz, t) \geq \mathcal{M}_2(Tz, w, w, t/q)$, which is a contradiction.

Thus $Tz = w$. Now we prove $Sw = z$. Suppose $Sw \neq z$. We have

$$\begin{aligned}
\mathcal{M}_1(Sw, z, z, qt) &= \lim_{n\to\infty} \mathcal{M}_1 (Sw , x_{n+1}, x_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_1 (Sw, STx_n, STx_n, qt) \\
&\geq \lim_{n\to\infty} \min\{\mathcal{M}_1(x_n, Sw, Sw, t) * \mathcal{M}_1 (x_n, STx_n, STx_n, t), \\
&\qquad \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_2(w, Tx_n, Tx_n ,t), \mathcal{M}_2(Tx_n, TSw, TSw, t)\}
\end{aligned}$$

$= \lim_{n\to\infty} \min\{\mathcal{M}_1(x_n, Sw, Sw, t) * \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t),$
$\mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_2(w, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_{n+1}, w, w, t)\}$
$\geq \mathcal{M}_2(w, TSw, TSw, t)$

Now, $\mathcal{M}_2(w, TSw, TSw, qt) = \lim_{n\to\infty} \mathcal{M}_2(y_{n+1}, TSw, TSw, qt) = \lim_{n\to\infty} \mathcal{M}_2(Tx_n, TSw, TSw, qt)$
$\geq \lim_{n\to\infty} \min\{\mathcal{M}_1(x_n, Sw, Sw, t), \mathcal{M}_1(Sw, STx_n, STx_n, t)$
$\mathcal{M}_2(w, Tx_n, Tx_n, t) * \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(x_n, STx_n, STx_n, t)\}$
$= \lim_{n\to\infty} \min\{\mathcal{M}_1(x_n, Sw, Sw, t), \mathcal{M}_1(Sw, x_{n+1}, x_{n+1}, t),$
$\mathcal{M}_2(w, y_{n+1}, y_{n+1}, t) * \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t)\}$
$\geq \mathcal{M}_1(z, Sw, Sw, t).$

Hence, $\mathcal{M}_1(Sw, z, z, qt) \geq \mathcal{M}_2(w, TSw, TSw, t) \geq \mathcal{M}_1(z, Sw, Sw, t/q)$. Which is a contradiction.

Thus Sw = z. we have STz = Sw = z and TSw = Tz = w.

Thus the point z is a fixed point of ST in X and the point w is a fixed point of TS in Y.

Uniqueness: Let $z' = z$ be the another fixed point of ST in X. We have
$\mathcal{M}_1(z, z', z', qt) = \mathcal{M}(Sw, STz', STz', qt)$
$\geq \min\{\mathcal{M}_1(z', SW, SW, t) * \mathcal{M}_1(z', STz', STz', t), \mathcal{M}_2(w, TSw, TSw, t),$
$\mathcal{M}_2(w, Tz', Tz', t), \mathcal{M}_2(Tz', TSw, TSw, t)\}$
$= \min\{\mathcal{M}_1(z', z, z, t) * 1, 1, \mathcal{M}_2(w, Tz', Tz', t), \mathcal{M}_2(Tz', w, w, t)\}$
$\geq \mathcal{M}_2(Tz', w, w, t)$

Now, $\mathcal{M}_2(Tz', w, w, qt) = \mathcal{M}_2(Tz', TSw, TSw, qt)$
$\geq \min\{\mathcal{M}_1(z', Sw, Sw, t), \mathcal{M}_1(Sw, STz', STz', t),$
$\mathcal{M}_2(w, Tz', Tz', t) * \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(z', STz', STz', t)\}$
$= \min\{\mathcal{M}_1(z', z, z, t), \mathcal{M}_1(z, z', z', t), \mathcal{M}_2(w, Tz', Tz', t) * 1, 1\}$
$\geq \mathcal{M}_1(z, z', z', t)$

Hence, $\mathcal{M}_1(z, z', z', qt) \geq \mathcal{M}_2(Tz', w, w, t) \geq \mathcal{M}_1(z, z', z', t/q)$. Which is a contradiction.

Thus $z = z'$. So the point z is a unique fixed point of ST.

Similarly, we prove the point w is also a unique point of TS.

Corollary 2.8: Let $(X, \mathcal{M}, *)$ be a complete $\mathcal{M}$-fuzzy metric space, If S and T are mapping from X into itself satisfying the following conditions.

$\mathcal{M}(Tx, TSy, TSy, t) \geq \min\{\mathcal{M}(x, Sy, Sy, t), \mathcal{M}(Sy, STx, STx, t),$
$\mathcal{M}(y, Tx, Tx, t) * \mathcal{M}(y, TSy, TSy, t), \mathcal{M}(x, STx, STx, t)\}$

$\mathcal{M}(Sy, STx, STx, t) \geq \min\{\mathcal{M}(x, Sy, Sy, t) * \mathcal{M}(x, STx, Stx, t),$
$\mathcal{M}(y, TSy, TSy, t), \mathcal{M}(y, Tx, Tx, t), \mathcal{M}(Tx, TSy, TSy, t)\}$

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

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