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CONTRIBUTIONS ON M-FUZZY FACTOR GROUP

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ABSTRACT

The purpose of this paper is to generalize new definition of upper M-fuzzy factor groups and using this definition to study some properties for this subject.

***Keywords**: M-fuzzy factor subset, M-fuzzy factor group, M-fuzzy factor normal subgroups.*

INTRODUCTION

The fundamental concept of fuzzy sets was initiated by Lofti Zadeh [12] in 1965 and opened a new path of thinking to mathematicians, engineers, physicists, chemists and many other due to its diverse application is various fields. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, coding theory, group theory, real analyses, hectare theory etc. In 1971, Rosenfeld [10] first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic structures. Thereafter the notion of different fuzzy algebraic structures such as fuzzy ideals in rings and semi rings etc, have seriously studied by mathematicians. In 1975, Zadeh [12] introduced the concepts of interval-valued fuzzy sets, where the values of member instead of the real points. His definition has been generalized by Anthony and Sherwood [1]. We introduced the concept of fuzzy normal subgroups also Mukherjee and Bhattacharya [9] studied the normal fuzzy groups and fuzzy cosets, on the other hand the notion of a fuzzy subgroup ablian group was introduced by Murali and Makamba [8]. Basic definitions can be obtained in [2, 3]. The purpose of this paper is to generalize new definition of upper M-fuzzy factor groups and using this definition to study some properties for this subject.

Definition 1.1: Let X be a set. Then a mapping μ: M $\times$X$\rightarrow$ [0, 1] is called a M- fuzzy subset of X.

Definition 1.3: Let "G" be any group. A mapping μ: G$\rightarrow$ [0, 1] is called a upper M-fuzzy group of G if
i) $\mu(mxy) \leq \max\{ \mu(x), \mu(y)\}$
ii) $\mu(mx^{-1}) \leq \mu(x)$ for allx, y ЄG

Definition 1.4: AM-fuzzy set A is called M-fuzzy group of G if
(QFG1): $A(mxy) \geq \text{Min}\,\{A(x),A(y)\}$
(QFG2): $A(mx^{-1}) = A(x,q)$
(QFG3): $A(mx) = 1$ for all x,yЄG and qЄG.

Proposition 1.1: If μ is a M-fuzzy group of a group G having identity e, then
i) $\mu(mx^{-1})=\mu(x)$
ii) $\mu(me) \leq \mu(x)$ $\forall$xЄG

Definition 1.5: Let μ be a M-fuzzy group of G. then "μ" is called a M-fuzzy normal group if μ (mxy) = μ (myx) $\forall$x, yЄG

Definition 1.6: Let X be a set. Then a mapping μ: M$\times$X$\rightarrow$ $M^*([0, 1])$ is called M-fuzzy factor subset of X, where $M^*([0, 1])$ denotes the set of all non empty subset of [0, 1].

Definition 1.7: Let X be a non empty set and μ and λ be two M-fuzzy factor subset of X. Then the intersection of μ and λ denoted by $\mu\cap\lambda$ and defined by

$\mu\cap\lambda = \{\min\{a, b\},/\ a\in\mu(mx),\ b\in\lambda(mx)\}$ for all $x\in X$ and $m \varepsilon M$.

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The union of μ and λ and denoted by $\mu\cup\lambda$ and defined by

$\mu\cup\lambda = \{\max\{a, b\}/ a\in\mu(mx), b\in\lambda(mx)\}$ for all $x\in X$ and $m \varepsilon M$

Definition 1.8: Let X be a groupoid that is a set which is closed under a binary relation denoted multiplicatively. A mapping is called upper M-fuzzy factor groupoid if for all x, y$\in$ x, following conditions hold:
(i) $\inf \mu(mxy) \le S\{\inf \mu(x), \inf \mu(y)\}$
(ii) $\sup \mu(mxy) \le S\{\sup \mu(x), \sup \mu(y)\}$

Definition1.9: Let G be a group. A mapping $\mu: G\rightarrow M^*([0, 1]$ is called a upper M-fuzzy factor group of G if for all x, y $\in$ G, following conditions hold:
(i) $\inf \mu(mxy) \le S\{\inf \mu(x),\inf \mu(y)\}$
(ii) $\sup \mu(mxy) \le S\{\sup \mu(x), \sup \mu(y)\}$
(iii) $\inf \mu(mx^{-1}) \le \inf \mu(x)$
(iv) $\sup \mu(mx^{-1}) \le \sup \mu(x)$

Note: In definition * if μ: G$\rightarrow$ [0, 1] then $\mu(x)$ $\forall x\in G$ are real points in [0, 1] and also $\inf(\mu(x)) = \sup \mu(x) = \mu(x)$, $x\in G$. Thus definition * reduces to definition of Rosen field's upper fuzzy groups. So Upper factor group is a generalization of Rosenfeld's fuzzy group.

Proposition 1.2: An upper M-fuzzy factor subset μ of a group G is a upper M- fuzzy factor group iff for all x, y$\in$G followings are hold.

(i) $\inf\mu(mxy^{-1}) < S \{\inf\mu(x), \inf \mu(y)\}$
(ii) $\sup\mu(mxy^{-1}) < S \{\sup\mu(x), \sup \mu(y)\}$

Proof: At first let μ be upper M-fuzzy factor group of G and x, y$\in$G. Then

$$\begin{aligned}\inf\mu(mxy^{-1}) &< S \{\inf\mu(x), \inf \mu(y^{-1})\}\\ &= S \{\inf\mu(x), \inf \mu(y)\} \text{ and}\end{aligned}$$

$$\begin{aligned}\sup\mu(mxy^{-1}) &< S \{\sup\mu(x), \sup \mu(y)\}\\ &= S \{\sup\mu(x), \sup \mu(y)\}\end{aligned}$$

Conversely, let μ be upper M-fuzzy factor subset of G and given conditions hold. Then for all x$\in$G, we have

$\inf \mu(me) = \inf \mu(mxx^{-1}) \le S\{\inf \mu(mx),\inf \mu(mx)\}= \inf \mu(mx)$ ______(1)

$\sup \mu(me) = \sup \mu(mxx^{-1}) \le S\{\sup \mu(x), \sup \mu(x)\}=\sup \mu(x)$ ______(2)

So, $\inf \mu(mx^{-1}) = \inf \mu(mex^{-1}) \le S\{\inf \mu(e), \inf \mu(x)\}= \inf \mu(x)$ by (1)
And
$\text{Sup } \mu(mx^{-1}) = \text{Sup } \mu(mex^{-1}) \le S\{\text{Sup } \mu(e), \text{Sup } \mu(x)\}= \text{Sup } \mu(x)$ by (2)

Again

$$\begin{aligned}\inf\mu(mxy) &\le S \{\inf\mu(x), \inf \mu(y^{-1})\}\\ &\le S \{\inf\mu(x), \inf \mu(y)\}\end{aligned}$$

And

$$\begin{aligned}\sup\mu(mxy) &\le S \{\sup\mu(x), \sup \mu(y)\}\\ &\le S \{\text{Sup}\mu(x), \text{Sup } \mu(y)\}\end{aligned}$$

Hence μ is a upper M-fuzzy factor group of G.

Proposition1.3: If μ is an upper M-fuzzy factor groupoid of an infinite group G, then μ is an upper M-fuzzy factor group of G.

Proof: Let x$\in$G. Since G is finite, x has finite order, say p. then x^p=e, where e is the identity of G. Thus $x^{-1}= \mu^{p-1}$using the definition of Upper M-fuzzy factor groupoid, we have

$\inf \mu(mx^{-1}) = \inf \mu(mx^{p-1}) = \inf \mu(mx^{p-2}) \le S\{\inf \mu(x^{p-2}), \mu(x)\}$

Again
$\inf \mu(mx^{p-2}) = \inf \mu(mx^{p-3},x) \le S\{\inf \mu(x^{p-3}), \mu(x)\}$

Then we have
$\inf \mu(mx^{-1}) \leq S\{\inf \mu(x^{p-3}), \inf \mu(x)\}$

So applying the definition of Upper r M-fuzzy factor groupoid repeatedly, we have that, $\inf \mu(mx^{-1}) \leq \inf \mu(x)$

Similarly we have, $\sup \mu(mx^{-1}) \leq \sup \mu(x)$

Therefore μ is an Upper M-fuzzy factor group.

Proposition 1.4: The Intersection of any two Upper M-fuzzy factor groups is also a Upper M-fuzzy factor group of G.

Proof: Let A and B be any two Upper M-fuzzy factor groups of G and x, y$\in$ G then
$\inf(A\cap B)(mxy^{-1}) = S\{\inf A(xy^{-1}), \inf B(xy^{-1})\}$
$\leq S\{S\{\{\inf A(x), \inf A(x)\}, \{\inf B(x), \inf B(y)\}\}\} S\{S\{\inf A(x), \inf B(x)\}, S\{\inf A(x), \inf B(y)\}\}\}$
$= S\{\inf A\cap B(x), \inf A\cap B(y)\}$ ____________(1)
Again
$\sup(A\cap B)(mxy^{-1}) = S\{\sup A(xy^{-1}), \sup B(xy^{-1})\}$ by definition
$\leq S\{S\{\{\sup A(x), \sup A(x)\}, \{\sup B(x), \sup B(y)\}\}\} S\{S\{\sup A(x), \sup B(x)\}, S\{\sup A(x), \sup B(y)\}\}\}$
$= S\{\sup A\cap B(x), \sup A\cap B(y)\}$ ____________(2)

Hence by (1) and (2) and using proposition we say

$A\cap B$ is Upper M-fuzzy factor group of G.

Proposition 1.5: If A is a upper M-fuzzy factor group of a group G having identity e, then$\forall x\in X$
i) $\inf A(mx^{-1}) = \inf A(x)$ and $\sup A(mx^{-1}) = \sup A(x)$
ii) $\inf Am(e) \leq \inf A(x)$ and $\sup A(me) = \sup A(x)$

Proof: (i) As A is a upper M-fuzzy factor group of a group G, Then $\inf A(mx^{-1}) \leq \inf A(x)$

Again $\inf A(mx) = \inf A(m(x^{-1})^{-1}) \leq \inf A(x^{-1})$

So $\inf A(mx^{-1}) = \inf A(x)$

Similarly we can prove that $\sup A(mx^{-1}) = \sup A(x)$

(ii) $\inf A(me) = \inf A(mxx^{-1}) \leq S\{\inf A(x), \inf A(x^{-1})\}$

And $\sup A(me) = \sup A(mxx^{-1}) \leq S\{\sup A(x), \sup A(x^{-1})\}$

Proposition 1.6: Let μ and λ be two upper M-fuzzy factor group of G_1, G_2 respectively and let Q be a homomorphism from G_1 to G_2. Then
(i) $Q(m\mu)$ is a upper M-fuzzy factor group of G_2
(ii) $Q(m\lambda)$ is a upper M-fuzzy factor group of G_1

Proof: It is trivial

Remark: If μ is upper M-fuzzy factor group of G and K is subgroup of G then the restriction of μ to $K(\mu/K)$ is upper M-fuzzy factor group of K.

2. UPPER NORMAL M-FUZZY FACTORGROUP

Definition2.1: If μ is an upper M-fuzzy factor group of a group G then μ is called a upper normal M-fuzzy factor group of G if for all x, y$\in$G
$\inf\mu(mxy) = \inf\mu(myx)$ and
$\sup\mu(mxy) = \sup\mu(myx)\}$

Proposition 2.1: The Intersection of any two Upper normal M-fuzzy factor groups of G is also a Upper normal M-fuzzy factor group of G.

Proof: Let A and B be any two Upper normal M-fuzzy factor groups of G. By proposition $A\cap B$ is an Upper M-fuzzy factor group of G.

Let x, y$\in$ G then by definition

$\inf(A\cap B)(mxy) = S\{\inf A(xy), \inf B(xy)\}$ by definition

$= S\{\inf A(yx), \inf B(yx)\}$

$= \inf A\cap B(yx)$

Similarly $\sup(A\cap B)(mxy) = \sup(A\cap B)(yx)$

This shows that A$\cap$B is Upper normal M-fuzzy factor group of G.

Proposition 2.2: The Intersection of any arbitrary collection of Upper normal M-fuzzy factor groups of a group G is also a Upper normal M-fuzzy factor group of G.

Proof: Let x, y$\in$ G and $\alpha \in$ G

$\inf A(mxy^{-1}) = \inf A(\alpha^{-1}xy^{-1}\alpha)$ by definition

$= \inf A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha)$

$= \inf (A(\alpha^{-1}x\alpha),A((\alpha^{-1}y\alpha)^{-1}))$

$\leq S\{\inf (A(\alpha^{-1}x\alpha),\inf A((\alpha^{-1}y\alpha)))\}$

$= S\{\inf (A(x),A((y))\}$

Again

$\sup A(mxy^{-1}) = \sup A(\alpha^{-1}xy^{-1}\alpha)$ by definition

$= \sup A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha)$

$= \sup (A(\alpha^{-1}x\alpha),A((\alpha^{-1}y\alpha)^{-1}))$

$\leq S\{\sup (A(\alpha^{-1}x\alpha), \sup A((\alpha^{-1}y\alpha)))\}$

$= S\{\sup (A(x),A((y))\}$

Hence by proposition 1.2 A is Upper normal M-fuzzy factor group of G.

CONCLUSION

In this paper, the concept of upper M-fuzzy factor set is introduced and thereafter we defined upper M-fuzzy factor group and a few of its properties are discussed.

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