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NUMERICAL STUDY OF SYNCHRONIZATION & ANTI- SYNCHRONIZATION OF A ROTATING SATELLITE

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ABSTRACT

This paper considers the synchronization and anti-synchronization (AS) behaviour of two identical nonlinear dynamical systems of a rotating satellite in elliptic orbit evolving from different initial conditions using the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The designed controller, with our own choice of the coefficient matrix of the error dynamics, are found to be effective in the stabilization of the error states at the origin, thereby, achieving synchronization and anti-synchronization between the states variables of two dynamical systems under consideration. Numerical simulations are presented to illustrate the effectiveness of the proposed control techniques.

Key words: *Chaos; Synchronization; Anti-synchronization (AS); satellite.*

1. INTRODUCTION

After the pioneering work on chaos control by Ott *et al* [1] and synchronization of chaotic systems by Pecora & Carroll [2], chaos control and synchronization has received a wide attention and has become a very active topic in nonlinear sciences since last couple of years. In this direction, various effective methods have been investigated to achieve the control and stabilization of chaotic systems in almost all fields of nonlinear sciences [3–15]. The OGY method, for instance, have been successfully applied to many chaotic systems like the pendulum dynamics [16, 17]. Also, the Pyragas time-delayed auto-synchronization method [18, 19] has been shown to be an efficient method that has been realized experimentally on the dynamics of oscillators, lasers and chemical systems [20-22].

In particular, backstepping design and active control have been recognised as two powerful design methods to synchronize the nonlinear dynamical systems. It has been reported [23–25] that backstepping design can guarantee global stability, tracking and transient performance for a broad class of strict-feedback nonlinear systems. In recent time, it has been employed for controlling, tracking and synchronizing many chaotic systems as well as hyperchaotic systems [26-32].

Motivated by aforementioned studies, we prepare the present article using the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria to study the synchronization and AS behavior of the two identical planar oscillation of a satellite in elliptic orbit evolving from different initial conditions.

2. EQUATION OF MOTION OF A SATELLITE

In the present work, we consider the planar oscillation of a satellite in elliptic orbit with the spin axis fixed perpendicular to the orbital plane. Let the long axis of the satellite makes an angle θ with a reference axis that is fixed in inertial space, the long axis of the satellite makes an angle ψ with satellites planet centre line [33] and the satellite to be a triaxial ellipsoid with principal moments of inertia $A < B < C,$ where C is the moment about the spin axis. The orbit is taken to be a fixed ellipse with semi major axis a, eccentricity e, true anomaly f, and instantaneous radius r. The equation of motion of satellite planar oscillation in an elliptic orbit around the earth is

$$2C\ddot{\theta} - 3(B-A)\frac{Gm_p}{r^3}\sin 2\psi = 0\,. \tag{2.1}$$

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Let $\dfrac{3(B-A)}{C}=\omega_0^2$, $Gm_p=\mu$, $\psi=f-\theta$ and using the relations $r^2\dot{f}=h$ and $\dfrac{l}{r}=1+e\cos f$, the equation (2.1) may be written as

$$\ddot{\theta}=\frac{\omega_0^2\mu}{2l^3}(1+e\cos f)^{-3}\sin 2(f-\theta). \tag{2.2}$$

3. SYNCHRONIZATION VIA ACTIVE CONTROL

The idea of synchronization of two identical chaotic systems that start from different initial conditions consists of linking the trajectory of one system to the same values in the other so that they remain in step with each other, through the transmission of a signal. For a system of two coupled chaotic oscillators, the master system ($\dot{x}=f(x,y)$) and the slave system ($\dot{y}=g(x,y)$), where $x(t)$ and $y(t)$ are the phase space (state variables), and $f(x,y)$ and $g(x,y)$ are the corresponding nonlinear functions, synchronization in a direct sense implies $|x(t)-y(t)|\to 0$ as $t\to\infty$. When this occurs the coupled systems are said to be completely synchronized. The problem may be treated as the design of control laws for full chaotic observer (the slave system) using the known information of the plant (the master system) so as to ensure that the controlled receiver synchronizes with the plant. Hence, the slave chaotic system completely traces the dynamics of the master in the course of time.

The system defined by (2.2) together with $r^2\dot{f}=h$, can be written as a system of three first order differential equations when the three variables are introduced:

$$\theta=x_1,\ \dot{x}_1=x_2,\ f=x_3.$$

The new system is:

$$\begin{aligned}
\dot{x}_1&=x_2,\\
\dot{x}_2&=\frac{\omega_0^2\mu}{2l^3}\left(1+e\cos x_3\right)^{-3}\sin 2(x_3-x_1),\\
\dot{x}_3&=\frac{h}{l^2}(1+e\cos x_3)^{-2}.
\end{aligned} \tag{3.1}$$

Let us define another system as follows:

$$\begin{aligned}
\dot{y}_1&=y_2+u_1(t),\\
\dot{y}_2&=\frac{\omega_0^2\mu}{2l^3}(1+e\cos y_3)^{-3}\sin 2(y_3-y_1)+u_2(t),\\
\dot{y}_3&=\frac{h}{l^2}(1+e\cos y_3)^{-2}+u_3(t).
\end{aligned} \tag{3.2}$$

where (3.1) and (3.2) are called the master and the slave systems respectively, and in the slave system $u_i(t)$ (for $i=1,2,3$) are control functions to be determined. Let $e_1=y_1-x_1$, $e_2=y_2-x_2$ and $e_3=y_3-x_3$ be the synchronization errors such that in synchronization state $\lim\limits_{t\to\infty}e_i(t)\to 0$ (for $i=1,2,3$). From (3.1) and (3.2), we obtain the error dynamics

$$\begin{aligned}
\dot{e}_1&=e_2+u_1(t),\\
\dot{e}_2&=\frac{\omega_0^2\mu}{2l^3}\left\{\left(1+e\cos y_3\right)^{-3}\sin 2(y_3-y_2)-\left(1+e\cos x_3\right)^{-3}\sin 2(x_3-x_2)\right\}+u_2(t),\\
\dot{e}_3&=\frac{h}{l^2}\left\{(1+e\cos x_3)^{-3}-(1+e\cos y_3)^{-3}\right\}+u_3(t).
\end{aligned} \tag{3.3}$$

The error system (3.3) to be controlled is a linear system with control inputs. Therefore we redefine the control functions such as to eliminate terms in (3.3) which cannot be expressed as linear terms in e_1, e_2 and e_3 as follows:

$$
\begin{aligned}
u_1(t) &= v_1(t),\\
u_2(t) &= -\frac{\omega_0^2\mu}{2l^3}\left\{\left(1+e\cos y_3\right)^{-3}\sin 2(y_3-y_2)-\left(1+e\cos x_3\right)^{-3}\sin 2(x_3-x_2)\right\}+v_2(t),\\
u_3(t) &= -\frac{h}{l^2}\left\{(1+e\cos x_3)^{-2}-(1+e\cos y_3)^{-2}\right\}+v_3(t).
\end{aligned}
\tag{3.4}
$$

Therefore the linear error system can be written as follows:

$$
\begin{aligned}
\dot{e}_1(t) &= e_2(t)+v_1(t),\\
\dot{e}_2(t) &= v_2(t),\\
\dot{e}_3(t) &= v_3(t).
\end{aligned}
\tag{3.5}
$$

The error system (3.5) to be controlled is a linear system with control inputs v_1, v_2 and v_3 as the function of the error states e_1, e_2 and e_3. As stated, as long as $\lim_{t\to\infty} e_i(t) \to 0$ for $i=1,2,3$, synchronization between the master and slave system is realized, that is, the system represented by (3.1) and (3.2) are synchronized under active control. According to active control method, the controllers v_1, v_2 and v_3 can be written as:

$$
\begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}^T = A\begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix}^T \tag{3.6}
$$

Where *A* is a 3×3 constant matrix. As per the Lyapunov stability theory and the Routh-Hurwitz criteria, in order to make the close loop system (3.5) stable, the proper choice of the elements of *A* is such that the system (3.6) must have all eigen values with the negative real parts.

Let $A=\begin{pmatrix} -\lambda_1 & -1 & 0\\ 0 & -\lambda_2 & 0\\ 0 & 0 & -\lambda_3 \end{pmatrix}$, with $\lambda_{i(i=1,2,3)} > 0$. In order to achieve the synchronization between (3.1) and (3.2), however we have many choices for λ_1, λ_2 and λ_3 but here we are choosing $\lambda_1=\lambda_2=\lambda_3=1$.

4. NUMERICAL SIMULATION FOR SYNCHRONIZATION

For the constant elements of feedback matrix, choosing $\lambda_1=\lambda_2=\lambda_3=1$ and the parameters involved in system under investigation, $e=0.15$, $h=0.1$, $l=0.7$, $\mu=0.02$ and $\omega_0=0.3$ together with the initial conditions for the master and slave systems $\left[x_1(0), x_2(0), x_3(0)\right]=\left[0, 0.1, 0.1\right]$ and $\left[y_1(0), y_2(0), y_3(0)\right]=\left[0.5, 0.9, 0.5\right]$, the system has been simulated using *mathematica*. The obtained results show that the system under consideration achieved synchronization. Time series analysis of the states variables involved in master & slave systems (Figures 1, 2 & 3) and time series analysis of errors (Figure 4) are the witness of achieving synchronization between master and slave system under consideration. Furthermore, it also has been confirmed by the convergence of the synchronization quality defined by

$$
e(t)=\sqrt{e_1^2(t)+e_2^2(t)+e_3^2(t)} \tag{4.1}
$$

Figure (9) confirms the convergence of the synchronization quality defined by (4.1).

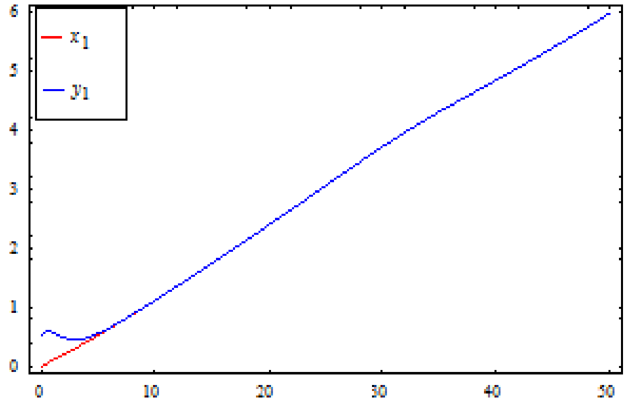

Figure 1: Time Series of x_1 & y_1

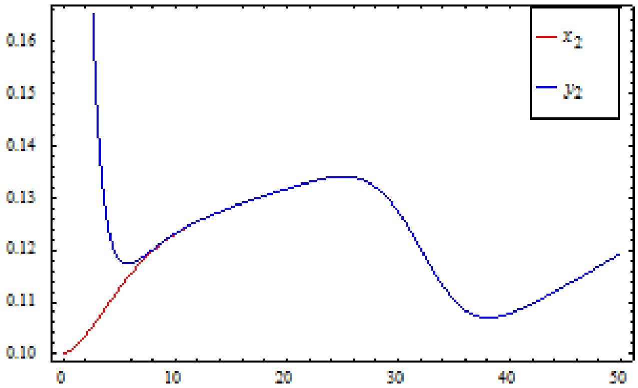

Figure 2: Time Series of x_2 & y_2

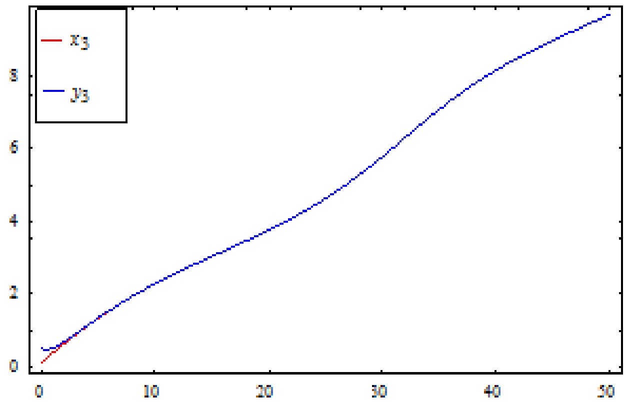

Figure 3: Time Series of x_3 & y_3

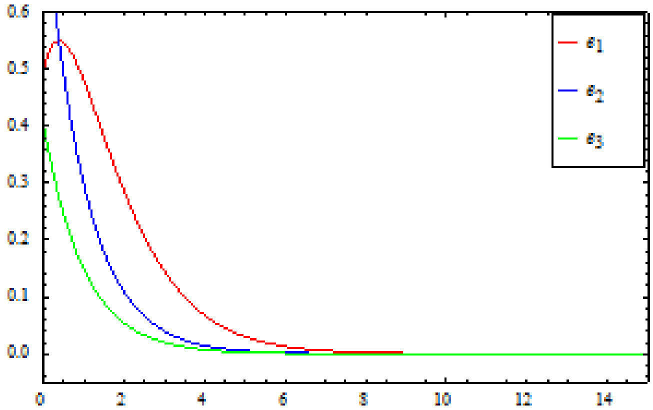

Figure 4: Time Series of errors in Synchronization

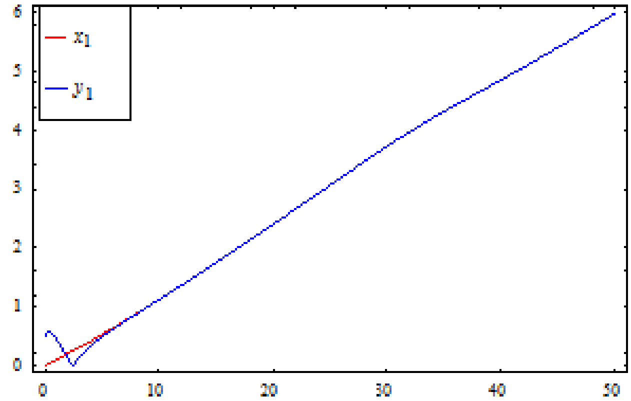

Figure 5: Time Series of x_1 & y_1 in AS

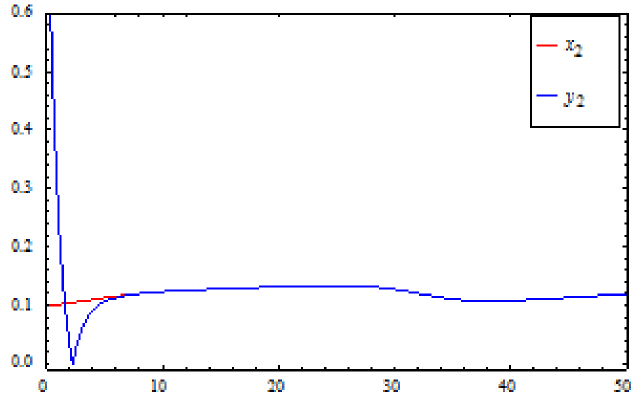

Figure 6: Time Series of x_2 & y_2 in AS

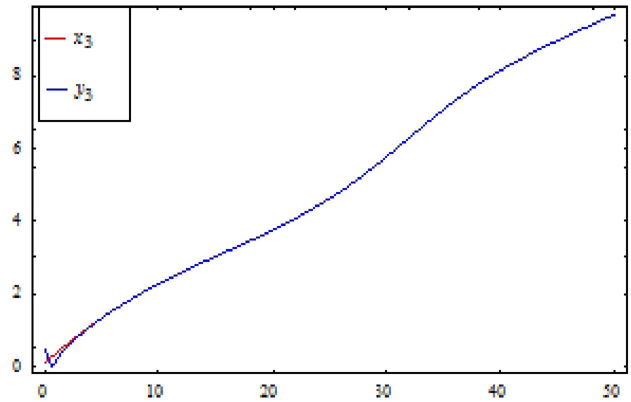

Figure 7: Time Series of x_3 & y_3 AS

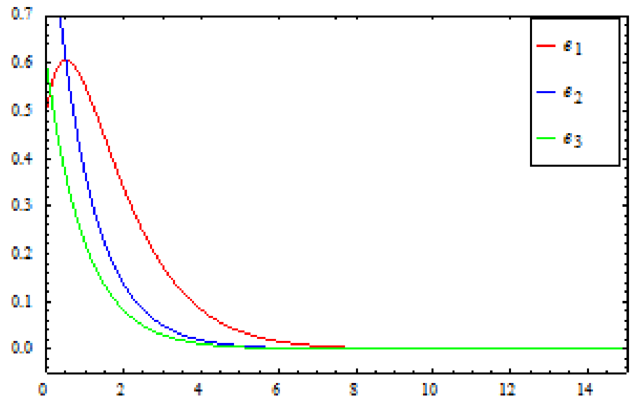

Figure 8: Time Series of errors in AS

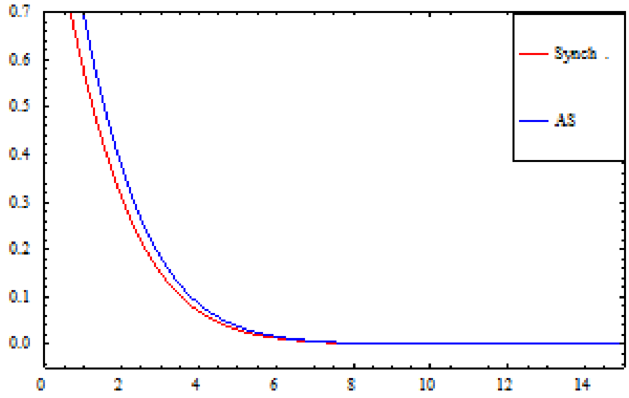

Figure 9: Convergence of errors

5. ANTI-SYNCHRONIZATION VIA ACTIVE CONTROL

When the state vectors of synchronized systems have the same absolute values but opposite signs i.e. the sum of the output signals of two systems can converge to zero, called AS. For two coupled systems: master ($\dot{x} = f(x, y)$) and slave ($\dot{y} = g(x, y)$) if $\lim_{t\to\infty} |x(t) + y(t)| \to 0$, the system under study has achieved the AS. This phenomenon has been investigated both experimentally and theoretically for many nonlinear dynamical systems [34-41].

In order to formulate the active controllers for AS, we redefine the error function as:

Let $e_1(t) = y_1(t) + x_1(t)$, $e_2(t) = y_2(t) + x_2(t)$ and $e_3(t) = y_3(t) + x_3(t)$ be the AS errors such that $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1, 2, 3$. From (3.1) and (3.2), we obtain the error dynamics for AS phenomenon as follows:

$$\begin{aligned}
\dot{e}_1 &= e_2 + u_1(t), \\
\dot{e}_2 &= \frac{\omega_0^2 \mu}{2l^3} \left\{ (1 + e\cos y_3)^{-3} \sin 2(y_3 - y_2) + (1 + e\cos x_3)^{-3} \sin 2(x_3 - x_2) \right\} + u_2(t), \\
\dot{e}_3 &= \frac{h}{l^2} \left\{ (1 + e\cos x_3)^{-2} + (1 + e\cos y_3)^{-2} \right\} + u_3(t).
\end{aligned} \tag{5.1}$$

The error system (5.1) to be controlled is a linear system with control inputs. Therefore we redefine the control functions such as to eliminate terms in (5.1) which cannot be expressed as linear terms in e_1, e_2 and e_3 as follows:

$$\begin{aligned}
u_1(t) &= v_1(t), \\
u_2(t) &= -\frac{\omega_0^2 \mu}{2l^3} \left\{ (1 + e\cos y_3)^{-3} \sin 2(y_3 - y_2) + (1 + e\cos x_3)^{-3} \sin 2(x_3 - x_2) \right\} + v_2(t), \\
u_3(t) &= \frac{h}{l^2} \left\{ (1 + e\cos x_3)^{-2} + (1 + e\cos y_3)^{-2} \right\} + v_3(t).
\end{aligned} \tag{5.2}$$

Therefore the linear error system can be written as follows:

$$\begin{aligned}
\dot{e}_1(t) &= e_2(t) + v_1(t), \\
\dot{e}_2(t) &= v_2(t), \\
\dot{e}_3(t) &= v_3(t).
\end{aligned} \tag{5.3}$$

The error system (5.3) to be controlled is a linear system with control inputs v_1, v_2 and v_3 as the function of the error states e_1, e_2 and e_3. As stated, as long as $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1, 2, 3$, AS between the driver and response system is realized, that is, the system represented by (3.1) and (3.2) are said to be anti-synchronized under active control. According to active control method, the controllers v_1, v_2 and v_3 can be written as:

$$\begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}^T = A \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix}^T \tag{5.4}$$

Where A is a 3×3 constant matrix. In order to make the close loop system (5.3) stable, the proper choice of the elements of A is such that the system (5.4) must have all eigen values with the negative real parts.

Let $A=\begin{pmatrix} -\lambda_1 & -1 & 0 \\ 0 & -\lambda_2 & 0 \\ 0 & 0 & -\lambda_3 \end{pmatrix}$, with $\lambda_{i(i=1,2,3)}>0$. In order to achieve the AS between (3.1) and (3.2), however we have many choices for λ_1,λ_2 and λ_3 but here we are choosing $\lambda_1=\lambda_2=\lambda_3=1$.

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$$e(t)=\sqrt{e_1^2(t)+e_2^2(t)+e_3^2(t)} \tag{6.1}$$

Figure (9) confirms the convergence of the AS quality defined by (6.1).

7. CONCLUSION

In this paper, we have investigated the synchronization and AS behaviour of the two identical planar oscillation of a satellite in elliptic orbit evolving from different initial conditions via the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The results were validated by numerical simulations using *mathematica*. For the errors in synchronization and AS behavior of the system, we have observed that all the errors are converging to zero (Figures 4 & 8). Furthermore the rate of convergence of errors in both synchronization and AS has been shown in figure 9.

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