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FUZZY ANALYTIC HIERARCHICAL PROCESS TO DETERMINE THE RELATIVE WEIGHTS IN MULTI-LEVEL PROGRAMMING PROBLEMS

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ABSTRACT

Multi-Level Programming (MLP) has been employed to organize and facilitate the procedure of analyzing great number of decision making situations. Analytic Hierarchy Process (AHP) is a powerful approach of hierarchical structure analysis and used here for weight the significance or importance of objective functions. However due to some intrinsic uncertainty and vagueness existing in the significance attributed to judgment of Decision Makers (DMs) requirements, the crisp pairwise comparisons in the conventional AHP seems to be insufficient. In this article, the main objective is to calculate and determine the relative weights the DMs objective functions by using a Fuzzy-AHP (FAHP) approach with different methods; eigen vector and geometric mean. Triangular and trapezoidal fuzzy numbers were introduced to improve the scaling scheme of Saaty's method. The approach allows DMs to provide their preferences that are upper and lower bounds of the variables they control. The hierarchical system then was converted into Scalar Optimization Problem (SOP) such that objective functions can be combined into a single one. A numerical illustrative example was presented.

Keywords: *Multi-level decision making; analytic hierarchy process; weighting approach; fuzzy numbers; Fuzzy-AHP.*

1. INTRODUCTION

Hierarchical decision making often involves many uncertain factors and it is hard to formulate. Contributions had been delivered by mathematicians, economists, engineers and many other researchers and developers. Bi-Level Programming (BLP), as a special case of MLP, was introduced by *Von Stackelberg* in the context of unbalanced economic markets. After that time this field has obtained a rapid development and intensive investigation in both theories and applications. Much effort has been done on the development of Multi-Level (ML) decision making modeling and solution approaches, but the study of MLP problems is not vast and wide as compared with BLP problems in the literature. Over the last three decades, tremendous amount of research effort has been made on MLP for hierarchical decentralized planning problems leading to the publication of many interesting results in the literature and many methodologies have been proposed to solve it potentially [1].

MLP problem can be defined as *p*-person, non-zero sum game with perfect information in which each player moves sequentially from top to bottom. MLP problem is characterized by a center that controls more than one independent division on the lower levels. For instance, by adopting three criteria with respect to; strategic, production and operational planning as objective functions for three different levels. MLP problems are difficult to solve and not every problem has a solution even though it has a nonempty compact feasible set and it have been proved to be ***NP***-hard because of its nonconvexity and nonuniqueness of lower levels optimal solutions even when all involved functions are linear [2].

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This article is organized as; section 2 gives an introduction of MLP problems, its characteristics, formulation and solution concept. AHP approach, representation of pairwise comparisons and uncertainty in AHP concepts are presented in section 3. Section 4 provides fuzzy preliminaries including fuzzy sets, fuzzy triangular and trapezoidal numbers. In section 5, FAHP approach for MLP problems, fuzzy scales tables, fuzzy membership functions and consistency test of the comparisons matrix. An illustrative example is presented in section 6. The article will be finalized with its discussion and conclusion.

2. MLP PROBLEMS

Hierarchical data structures are very common in the social and behavioral sciences and MLP are developed for analyzing hierarchically structured data. So, MLP is an important branch of Operation Research, that is consists of two or more levels, namely; first level, second level, and so on up to last level. MLP problem is a sequence of many optimization problems in which the constraints region of Decision Maker (DM) is determined by the solution of other DMs. The first (higher, upper) level Decision Make r (DM1) is called the center (leader). The lower levels Decision Makers (DM2, DM3, …) called followers. They execute their policies after the decision of higher levels DMs and then DM1 optimizes his objective independently but may be affected by the reaction of the followers. ML decision making models are used for representing many hierarchical optimization situations in real word strategic, planning, and management such asfinancial control, economic analysis, facility location, government regulation, organ izational management, Conflict resolution, network design, traffic assignment, signal optimization, planning for resource management, defense, transportation, central economic planning at the regional or national level to create model problems concerning organizational design [3].

2.1 MLP PROBLEM CHARACTERIZING

Followers always try to optimize their objective functions, so, they can take part of the system decision which be concerned by their control variables but they must take the goal or preference of the DM1 into consideration. DM1 defines his objective function and decision variables, this information then constrains the DM2's feasible space and so on. The preference information is delivered from the upper levels to the lower levels sequentially. The geometric properties of the linear MLP problems are obtained and shown that when all the functions of the MLP problem are linear and its feasible region is a polyhedron, the optimal solution occurs at a vertex of feasible region. MLP is particularly appropriate for problems with the following characteristics [4, 6]:

- The system has interactive decision making units within a predominantly hierarchical structure.
- The external effect on a DM's problem can be reflected in both his objective function and his set of feasible decisions.
- The loss of cost of one level is unequal to the added gain to other level.
- The order of the play is very important and the choice of the upper level limits affects the choice or strategy of the lower levels.
- The execution of decision is sequential, from upper to lower levels.
- Each DM controls only a subset of the decision variables.
- Each level optimizes its own objective function independently apart from other levels.
- Each DM is fully informed about all prior choices.

2.2 MLP PROBLEM FORMULATION AND SOLUTION CONCEPT

MLP problem's formulation has different versions that are given in many articles; it can be formulated as follows [7]:

$$
\begin{aligned}
&\max_{x_1}\; f_1(X) = c_{11}x_1 + c_{12}x_2 + \ldots + c_{1p}x_p, \text{ where } x_2 \text{ solves:}\\
&\max_{x_2}\; f_2(X) = c_{21}x_1 + c_{22}x_2 + \ldots + c_{2p}x_p, \text{ where } x_p \text{ solves:}\\
&\qquad\qquad \ldots\\
&\max_{x_p}\; f_p(X) = c_{p1}x_1 + c_{p2}x_2 + \ldots + c_{pp}x_p\\
&\qquad \text{s.t.}\quad A_1x_1 + A_2x_2 + \ldots + A_px_p \le b,\\
&\qquad\qquad\qquad x_1, x_2, \ldots, x_p \ge 0.
\end{aligned}
\tag{1}
$$

Where, $X=(x_1, x_2, ..., x_p)$ denote the decision variables under control of DM1, DM2, … and last DM respectively. For i =1, 2, …, p, x_i is n_i-dimensional decision variable, and $f_i(X)$ is the related objective function to 1st, 2nd, …, pth level, respectively. Let $X = x_1 \cup x_2 \cup ... \cup x_p$ and $n = n_1 + n_2 + \ldots + n_p$ then, $c_{11}, c_{21}, \ldots, c_{p1}$ are constant row vectors of size

$(1\times n_1)$, $c_{12}, c_{22}, \ldots, c_{p2}$ are of size $(1\times n_2)$ and $c_{1p}, c_{2p}, \ldots, c_{pp}$ are of size $(1\times n_p)$, b is an m-dimensional constant column vector, and A_i is an $m \times n_i$ constant matrix. Each DM has to improve his strategy from a jointly dependent set S ;

$$S = \{X \mid A_1x_1+A_2x_2 + \ldots + A_px_p \le b,\ x_1, x_2,\ldots, x_p \ge 0\}. \tag{2}$$

Solution approaches can be classified into four categories; extreme point search, transformation approach, descent&heuristic and evolutionary approach [2]. While, in [8] an additional category is added, interior points approach through the neural network approaches. According to the stages of development, approaches can be classified into only two categories; 1st category, extreme point search, transformation approach, and descent and heuristic can be referred to as the traditional approaches, and 2nd category, intelligent computation or evolutionary approach and interior point approach are based on more recent developments. DMs may require approximate optimal solutions, so, fuzzy concept is adopted for obtaining non-dominated solutions using fuzzy membership functions and the tolerance concept which simplifies the representation and the computations for the compromises among levels. Effective fuzzy methods are developed in literature, see, [9-16]. Interactive procedures have met with a great success with such situations include full cooperation among DMs without predetermined preference information, by using interactive and fuzzy interactive methods, MLP problem can be solved, giving the best satisfactory results. The basic concept is that the computational complexity with re-evaluation of the problem repeatedly by redefining the elicited membership functions values in the solution search process for searching higher degree of satisfaction and obtaining the satisfactory solutions, see [17-21].

The earliest work in FAHP appeared in [22], which utilized Triangular Fuzzy Numbers (TFNs) to model the pairwise comparisons made in order to elicit weights of preference of the decision alternatives considered. Since then, FAHP-related developments have been repeatedly reported in the concomitant literature; e.g., spatial allocation within FAHP [23], the approach of FAHP and fuzzy multiple-criteria decision-making [24], deriving priorities from FAHP [25] and revisiting the original FAHP [26]. Among the different methodologies used, it has been observed that FAHP approach was used extensively in decision making. The approach was used to select the best bridge construction method among the alternatives avoiding the inconsistency there in [27]. In the literature, FAHP has been widely used in solving many complicated decision making problems. FAHP and its extensions were developed in selecting the key capabilities in technology management [28]. The FAHP approach was used in the evaluation of computer integrated manufacturing alternatives. The same approach was used in the selection of the best location for a facility and in the evaluation of catering firms in Turkey [29]. Fuzzy integrated analytic hierarchy process approach is used for selecting strategic big-sized R&D programs in the sector of energy technology development [30]. It is further used it in multi-criteria supplier evaluation and vendor selection [31, 32]. Many researchers who have studied FAHP provided evidences that it shows more efficiency in handling human judgments than the classical AHP approach [33-35].

3. ANALYTIC HIERARCHY PROCESS (AHP)

AHP is a mathematical technique developed for incorporating multi criteria decision making and designed to solve its complex and obtaining the weights. The AHP which is a powerful tool in decision making situations was introduced and developed by Saaty in the 1980s. It is a systematic decision making method which includes both qualitative and quantitative techniques. In AHP, the decision making process starts with dividing the problem into a hierarchy of issues which should be considered in the work. These hierarchical orders help to simplify the illustration of the problem and bring it to a condition which is more easily understood. In each hierarchical level the weights of the elements are calculated. The decision on the final goal is made considering the weights. AHP uses pairwise comparisons matrices for determining the scores of alternatives with respect to a given criterion, or determining values of a weight vector. While, the majority practitioner agreed to use the effective mathematical technique, eigenvector method proposed in [36], some researchers suggested other choices such as mean transformation, or row geometric mean.

3.1 REPRESENTATION OF PAIRWISE COMPARISONS

By using pairwise comparisons process, weights or priorities are derived from a set of judgments. While it is difficult to justify weights that are arbitrarily assigned, it is relatively easy to justify judgments and the basis (hard data, knowledge, experience) for the judgments. A complete pairwise comparisons matrix A can be expressed as; $A=[a_{ij}]$ is a $(n\times n)$ matrix with the following properties; $a_{ij}>0$, $a_{ii}=1$, and $a_{ij}=1/a_{ji}$ for $i,j\in\{1, 2,\ldots, n\}$. The following comparisons matrix defined by Saaty, employs 1-9 scales that illustrated in Table 1.

$$A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \ldots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \ldots & \frac{w_2}{w_n} \\ \vdots & \vdots & & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \ldots & \frac{w_n}{w_n} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \tag{3}$$

Table 1: Saaty's scales for pairwise comparisons.

Verbal judgment	*Intensity of significance*
Equal	1
Marginal	3
Strong	5
Very strong	7
Extreme	9
Intermediate judgment values for fuzzy inputs	2, 4, 6, 8

The process requires each DM to provide judgments about the relative significance of his objective and then specify the preference for each other's. After the normalized matrix of pairwise comparisons matrix *A* for a hierarchical structure is designed, the normalized principal eigen vector (priority vector) can be obtained by some ways such as averaging across the rows.

3.2 UNCERTAINTY IN AHP

The conventional AHP approach is incapable of handling the uncertainty and vagueness involved in the mapping of DM's preference to an exact number or ratio. The major difficulty with classical AHP is its inability in mapping human judgments. In recent years it has been observed that due to confusion in DMs mind probable deviations should be integrated to the decision making process [37]. According to some natural uncertainty in the AHP, some researchers have used the fuzzy concept to fuzzify the method while other researchers are against it. Some of the researchers believe that the Saaty's AHP approach has some weaknesses. Saaty, 2008 believes that some uncertainty is lying in the nature of AHP approach. [38] have mentioned, the uncertainty associated with the mapping of decision makers judgment to number, is not taken into account by the AHP and also the preference and personal judgment of decision maker have huge effect on the AHP result.

In order to overcome these problems, some researchers and authors made the Saaty's AHP modified and fuzzified to formulate and control the uncertainty. Also in [33] has raised questions about certainty of the comparisons ratios used in the AHP. He had considered a situation in which the DM can express feelings of uncertainty while he is ranking or comparing different alternatives or criteria. The method he has used to take uncertainties into account is using fuzzy numbers instead of crisp numbers in order to compare the significance between the alternatives or criteria. He has considered trapezoidal membership function for comparisons ratios in AHP and in [34] has developed a new approach for triangular case. In fact they believe having a defined uncertainty in the form of fuzzy numbers in comparisons ratios and then obtaining the weights in fuzzy forms, help DMs to get a better understanding of the final significance of the factors and the uncertainty lying within them. However, Saaty, the developer of the AHP, is against fuzzification of his method. He believes the AHP by pairwise comparisons matrix is already fuzzy because some uncertainty lies in the nature of the method, i.e. he believes that ratios in the method are not absolute or crisp numbers, in fact they are fuzzy numbers, so making the AHP fuzzier not only does not guarantee better results but could make it worse [39].

4. FUZZY PRELIMINARIES

4.1 FUZZY SET AND FUZZY NUMBER

A major contribution of fuzzy set theory is its capability of representing vague data. Fuzzy set theory is an extension of classical set theory where elements have degrees of membership. A fuzzy set is a class of objects with a membership function ranging between zero and one. It was specifically designed to mathematically represent uncertainty and vagueness. The usual Arithmetic operations on real numbers can be extended to the ones defined on fuzzy numbers by means of Zadeh's extension principle. The benefit of extending crisp theory to fuzzy techniques is the strength in solving real-world problems, which inevitably entail some degree of imprecision in the variables and parameters measured and processed for the application [31]. The fuzzy numbers are convex and normalized fuzzy sets used in connection with applications where an explicit representation of the ambiguity and uncertainty found in numerical data is desirable. Among the commonly used fuzzy numbers which can be found frequently in literature such as [40-42] and which facilitates fuzzy arithmetic calculations, triangular and trapezoidal fuzzy numbers are likely to be the adoptive due to their simplicity in modeling easy interpretations.

Definition 1: A fuzzy number $\tilde{M}$ is convex and normalized fuzzy set on the real line R if:

1- There exists at least one $x_0 \in R$ such that $\mu(x_0) = 1$,
2- $\mu(x)$ is piecewise continuous,
3- For any $\alpha \in [0, 1]$, $\tilde{M}_{\alpha} = (x, \mu_{\alpha}(x) \geq \alpha)$ is a closed interval.

4.2 TRIANGULAR FUZZY NUMBER

The TFN is a special class of fuzzy number whose membership is described by three real numbers.

Definition 2: A TFN $\widetilde{M}$ on R is defined by a triple (α, β, γ) if its membership $\mu_\alpha(x)$: $R \to [0, 1]$ is equal to;

$$\mu(x) = \begin{cases} (x-\alpha)/(\beta-\alpha), & x\in[\alpha, \beta], \\ (\gamma- x)/(\gamma-\beta), & x\in[\beta, \gamma], \\ 0, & otherwise. \end{cases} \tag{4}$$

Fig. 1, displays the structure of a TFN $\widetilde{M}$ where, $\alpha\leq \beta\leq \gamma$ and α,γ stand for the lower and upper value of the support of $\widetilde{M}$ respectively, and β for the modal value. When $\alpha=\beta=\gamma$ it is a nonfuzzy number by convention. The basic operational laws between two TFNs $\widetilde{M}_1$ and $\widetilde{M}_2$ are as follows and also in case of trapezoidal numbers, these operation laws will be needed in order to able to estimate weights out of the fuzzy matrix $\widetilde{A}$.

- $\widetilde{M}_1 \oplus \widetilde{M}_2 = (\alpha_1+ \alpha_2, \beta_1+ \beta_2, \gamma_1+ \gamma_2)$. (5)
- $\widetilde{M}_1 \otimes \widetilde{M}_2 = (\alpha_1\, \alpha_2, \beta_1\, \beta_2, \gamma_1\, \gamma_2)$. (6)
- $\widetilde{M}^{-1} = (\alpha, \beta, \gamma)^{-1} \approx (\gamma_{ij}^{-1}, \beta_{ij}^{-1}, \alpha_{ij}^{-1})$. (7)
- $k\widetilde{M} = k(\alpha, \beta, \gamma) = (k\alpha, k\beta, k\gamma)$ for $k \geq 0$. (8)
- $\widetilde{M}_1 \oslash \widetilde{M}_2 = (\alpha_1/\gamma_2, \beta_1/\beta_2, \gamma_1/\alpha_2)$. (9)

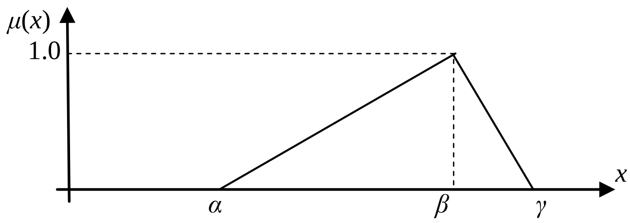

Fig. 1: Triangular fuzzy number

Definition 3: Let $\widetilde{M} = (\alpha, \beta, \gamma)$ be in Fuzzy$\in$ (R) then,

- $\widetilde{M}$ is said to be positive if $\alpha, \beta, \gamma \geq 0$,
- $\widetilde{M}$ is said to be integer if $\alpha, \beta, \gamma \geq 0$, and integers,
- $\widetilde{M}$ is said to be symmetric if $\beta - \alpha = \gamma - \beta$.

4.3 TRAPEZOIDAL FUZZY NUMBER

Trapezoidal membership functions can be also adopted for fuzzy numbers. In specific cases, by selecting accordingly the parameters of membership function, trapezoidal shape is converted to triangular. Trapezoidal fuzzy number's membership is defined by four real numbers and it can be defined by a quadruple ($\alpha, \beta, \gamma, \delta$) to approach the fuzziness of estimation, as follows:

$$\mu(x) = \begin{cases} (x-\alpha)/(\beta-\alpha), & x\in[\alpha, \beta], \\ 1, & x\in[\beta, \gamma], \\ (\delta - x)/(\delta-\gamma), & x\in[\gamma, \delta], \\ 0, & otherwise. \end{cases} \tag{10}$$

Where $\alpha \leq \beta \leq \gamma \leq \delta$ are the four parts of the fuzzy number, the membership function for a given fuzzy number, $\widetilde{M}$ is shown by $\mu(x)$ and these two numbers form an ordered pair $(x, \mu(x))$. Fig. 2 represents a fuzzy membership function with a trapezoidal fuzzy number.

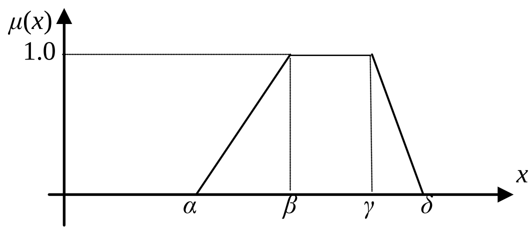

Fig. 2: Trapezoidal fuzzy number

The matrix $\tilde{A}$ means that it is fuzzy, then, $\tilde{A}= [\tilde{\alpha}_{ij}]$ where, $\tilde{\alpha}_{ij}= (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$ which indicates a trapezoidal membership function.

5. FAHP APPROACH FOR MLP PROBLEMS

AHP approach allows DMs to provide their assigned significance for their objective functions. In this article, a FAHP approach is presented, to deal with vagueness of human thought and in order to take the imprecision of human qualitative assessments into consideration. Then, the hierarchical system will be converted into SOP while objective functions can be combined into a single objective, where its weights represent the relative significance of DMs's objective functions. Here, DMs have other advantage that is their preferences bound for the decision variables that are the upper and lower bounds. First task of FAHP approach is to decide on the relative significance of each pair of objectives in MLP problem. Using fuzzy numbers, via pairwise comparisons, the fuzzy evaluation matrix $\tilde{A}= [\tilde{\alpha}_{ij}]$ is constructed and then, fuzzy pairwise comparisons are done using fuzzy linguistic preference variables for the estimation of the significance of MLP objectives.

5. 1 FUZZY SCALES

Fuzzification means using fuzzy numbers instead of the exact or crisp numbers and the meaning of membership scales can also be expressed by the commonly used fuzzy numbers; triangular or trapezoidal shapes in the same way as Saaty's scale. Let $\tilde{A}$ represents a fuzzified reciprocal ($n\times n$) judgment matrix containing all pairwise comparisons $\tilde{\alpha}_{ij}$ between elements i and j $\forall i, j\in\{1, 2, \ldots, n\}$. Fuzzy comparisons matrix $\tilde{A}$ differs with Saaty's in that using 0-1 membership scales by some ways instead of the 1-9 scales as the values of the elements.

Experts usually use the linguistic variable to evaluate the significance of the objectives and to rate its weights. In order to illustrate the idea of fuzzy scales, [40] transformed the existing precise values to five levels of fuzzy linguistic variables from 0.1 to 0.9; very low, low, medium, high, and very high, and [41] presented five levels also but from 1 to 9; equally, moderately, strongly, very strongly, and extremely,. While, [42] used six levels from 1 to 6; just equal, low, medium low, medium high, high, and very high and others. In this article, considering three calculation methods to obtain the fuzzy weights for MLP problem's objective as described in Section 5.3. The proposed fuzzy scales transform the existing precise values to five levels; extremely insignificant, insignificant, equally significant, moderately significant, and extremely significant as in Table 2.

Table 2: Fuzzy scales for pairwise comparisons

Verbal judgment	Explanation	Intensity of significance	
		Triangular fuzzy number	Narrow trapezoidal fuzzy number
Extremely insignificant	An objective is strongly inferior to another	(0.00, 0.10, 0.25)	(0.00, 0.05, 0.15, 0.25)
Insignificant	An objective is slightly inferior to another	(0.15, 0.30, 0.45)	(0.15, 0.25, 0.35, 0.45)
Equally significant	Two objectives contribute equally to the object	(0.35, 0.50, 0.65)	(0.35, 0.45, 0.55, 0.65)
Moderately significant	Judgment slightly favor one objective over another	(0.55, 0.70, 0.85)	(0.55, 0.65, 0.75, 0.85)
Extremely significant	Judgment strongly favor one objective over another	(0.75, 0.90, 1.00)	(0.75, 0.85, 0.95, 1.00)

5.2 FUZZY MEMBERSHIP FUNCTIONS

As a rule of thumb and as shown in Table 2, each rank is assigned an evenly spread membership function that has an interval of 0.25 or 0.30. For example, fuzzy variable, extremely insignificant has its associated TFN with the triple (α, β, γ) = (0.00, 0.10, 0.25) and associated trapezoidal fuzzy number with the quadruple ($\alpha, \beta, \gamma, \delta$) = (0.00, 0.05, 0.15, 0.25). In order to take the imprecision of human qualitative assessments into consideration, Fig. 3, illustrates the triangular fuzzy membership function for fuzzy variables. The highest density of the possibility of the fuzzy number is collected in triangular case while the difference Δ between β_{ij} and γ_{ij} equals to zero ($\beta_{ij} = \gamma_{ij}$) and by separating these two variables it is possible to spread the distribution of the fuzzy number in order to add to uncertainty. As soon as Δ zero, it gives more uncertainty and larger Δ, results in higher uncertainty. Trapezoidal fuzzy numbers can be described as narrow, medium and wide shapes with small, medium and large values of Δ respectively, such as (Δ=1, 2, 3).

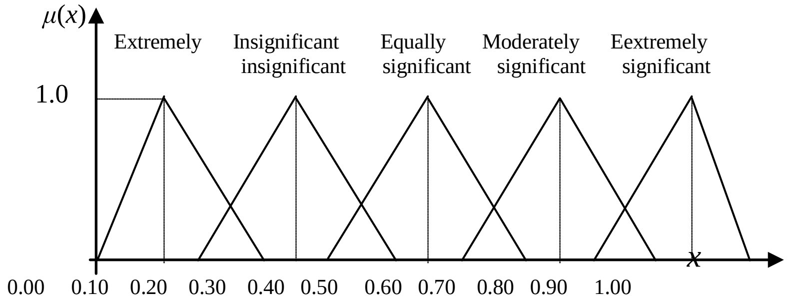

Fig. 3: Fuzzy triangular membership function

Fig. 4, illustrates the presented narrow trapezoidal fuzzy membership function for fuzzy variables; extremely insignificant, insignificant, equally significant, moderately significant, and extremely significant as described in Table 2, medium and wide cases can be described in the same manner.

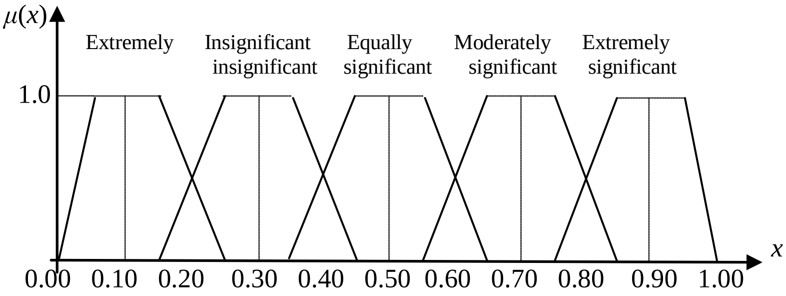

Fig. 4: Fuzzy narrow trapezoidal membership function

TFN can be expressed as a quadruple form such as (α_{ij}, β_{ij}, β_{ij}, γ_{ij}) and each pairwise comparisons matrix is fuzzy positive reciprocal ($n\times n$) matrix with elements $\tilde{a}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$ where;

$$\tilde{a}_{ji} = (\tilde{a}_{ij})^{-1} = (\delta_{ij}^{-1}, \gamma_{ij}^{-1}, \beta_{ij}^{-1}, \alpha_{ij}^{-1}), \text{ for all } i,j\in\{1, 2, \ldots, n\}. \tag{11}$$

5.3 CONSISTENCY TEST OF THE FUZZY COMPARISONS MATRIX

The theory of AHP does not demand perfect consistency and allows some small inconsistency in judgment and provides a measure of inconsistency because human is not always consistent. Consistency Ratio, (CR) must be checked before fuzzifying the pairwise comparisons matrix followed by calculating the corresponding weights.

Theorem 1: [33] Consider $\tilde{A} = [\tilde{a}_{ij}]$ where, $\tilde{a}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$ and let $\beta_{ij} \le \alpha_{ij} \le \gamma_{ij}$ for all i,j ; If $A = [a_{ij}]$ is consistent then, $\tilde{A}$ is also consistent.

Definition 4: A feasible solution $X^*\in S$ is a non-dominated solution if there does not exist any other feasible solution $X\in S$ such that $f_i(X^*) \le f_i(X)$, $\forall i\in\{1, 2,.., n\}$ and $f_i(X^*) < f_i(X)$, for at least one i.

Given the above-mentioned fuzzy preliminaries, the presented FAHP procedure of solving MLP problems is then defined in 6 steps as follows:

Step 1: Determination the problem inputs; intensity of significance of objectives and bounds of variables, constructing the pairwise comparisons matrix.

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \tag{12}$$

Where a_{ij} is the numerical answer given by each DM for the question "How many times objective i is more significant than objective j?"

Step 2: The comparisons matrix has to be normalized into the range of [0, 1], $N = [r_{ij}]$ by equation (13) and doing the consistency test where, perfect consistency implies a value of zero for Consistency Index (CI) $=(\lambda_{max} - n)/n-1$.

Therefore, it is considered acceptable if CI ≤ 10%. For CI values greater than 10%, the pairwise judgments may be revised before the weights are computed.

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \tag{13}$$

Step 3: Using triangular or/trapezoidal fuzzy numbers for transforming into fuzzy pairwise comparisons matrix, $\tilde{A}$ according to the mentioned values in Table 2.

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \tilde{a}_{nn} \end{bmatrix} \tag{14}$$

Step 4: Obtaining the FAHP approach outputs by calculating the relative weights for each objective throughout two methods by eigen vector and eigen value; λ_{max} and other method using geometric mean as follows:

a) After setting the degrees of fuzziness, transform into fuzzy environment using TFNs. Then, constructing the fuzzy pairwise comparisons matrix, $[\tilde{a}_{ij}]$ where $\tilde{a}_{ji} = (\tilde{a}_{ij})^{-1} = (\gamma_{ij}^{-1}, \beta_{ij}^{-1}, \alpha_{ij}^{-1})$ for all $i,j \in \{1, 2, \dots, n\}$. Normalizing the fuzzy pairwise comparisons matrix, $[\tilde{r}_{ij}]$ using the equation;

$$\tilde{r}_{ij} = \frac{\tilde{a}_{ij}}{\sum_{i=1}^{3} \tilde{a}_{ij}} \tag{15}$$

Where, for each resulting TFN component, $\alpha_{ij}, \beta_{ij}, \gamma_{ij} > 0$, $\sum_{i=1}^{3} \alpha_{ij} = 1$, $\sum_{i=1}^{3} \beta_{ij} = 1$, $\sum_{i=1}^{3} \gamma_{ij} = 1$, $\forall i,j \in \{1, 2, 3\}$. Once the normalized fuzzy pairwise comparisons matrix is completed, the corresponding fuzzy weights can be calculated with TFNs $\forall i \in \{1, 2, 3\}$ using the row average approach.

b) After constructing the pairwise comparisons matrix and its normalization matrix, the concept of fuzzy membership function can be applied to transform last matrix into fuzzy pairwise comparisons matrix using fuzzy linguistic variables and presented fuzzy scales with TFNs. Then, the corresponding fuzzy weights can be calculated by using the row average approach.

c) After the fuzzy comparisons matrix is defined, the normalization of the geometric mean method [33] is applied to compute the fuzzy weights. If $\tilde{a}_i$ is the geometric mean of i^{th} objective, $\tilde{a}_{ij}$ is the fuzzy comparisons value of objective i to objective j and $\tilde{w}_i$ is the i^{th} objective's weight.

$$\sum_{i=1}^{n} \tilde{w}_i = 1, \tilde{w}_i > 0. \tag{16}$$

$$\tilde{W}_i = \frac{\tilde{a}_i}{\sum_{i=1}^{n} \tilde{a}_i}, \text{ where } \tilde{a}_i = \left(\prod_{j=1}^{n} \tilde{a}_{ij}\right)^{1/n} \tag{17}$$

***Step** 5:* Providing the relative weights, constructing and formulating the final problem model where, MLP problem (1) can be stated with upper and lower bounds of decision variables; U_n and L_n as follows:

$$\max \; (\tilde{w}_1 f_1(X) + \tilde{w}_2 f_2(X) + \dots + \tilde{w}_n f_n(X)),$$

$$\begin{aligned} \text{s.t.} \quad & A_1x_1 + A_2x_2 + \dots + A_nx_n \le b, \\ & L_1 \le x_1 \le U_1, \\ & L_2 \le x_2 \le U_2, \\ & \dots \\ & L_n \le x_n \le U_n, \\ & x_1, x_2, \dots, x_n \ge 0. \end{aligned}$$

U_n and U_n and L_n are the upper and lower bounds of decision variables provided by the respective DM although, FAHP approach does not require any assumptions or information regarding DMs utility function.

Step 6: Solving the problem formed in step 5 to get out the optimal solution for the original MLP problem.

By solving the final problem's model, the weighting coefficients convey the significance attached to objective functions and the preferred solution is obtained.

6. ILLUSTRATIVE EXAMPLE

Consider the following numerical TLP example [11];

$$\max_{x1} f_1(x_1, x_2, x_3) = 7x_1 + 3x_2 - 4x_3 \text{, where, } x_2, x_3 \text{ solve:}$$

$$\max_{x2} f_2(x_1, x_2, x_3) = x_2 \text{, where, } x_3 \text{ solves:}$$

$$\max_{x3} f_3(x_1, x_2, x_3) = x_3 \tag{18}$$

$$\begin{aligned} \text{st:} \quad & x_1 + x_2 + x_3 \le 3, \\ & x_1 + x_2 - x_3 \le 1, \\ & x_1 + x_2 + x_3 \ge 1, \\ & -x_1 + x_2 + x_3 \le 1, \\ & x_3 \le 0.5, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

Once a series of questions were asked to direct pairwise comparisons, the original pairwise comparisons matrix with crisp ratios and its normalized matrix are performed as in Tables 3 and 4.

Table 3: Pairwise comparisons matrix for crisp ratios.

	f_1	f_2	f_3
f_1	1	2	4
f_2	0.5	1	1
f_3	0.25	1	1

Table 4: Normalized pairwise comparisons matrix with crisp ratios.

	f_1	f_2	f_3
f_1	0.57	0.50	0.66
f_2	0.29	0.25	0.17
f_3	0.14	0.25	0.17

It easy to see that; the principal eigen value; λ_{max}=3.055. Then, CI=0.0275 and the Random Consistency Index (RI)=0.58. Then, CR=4.74%≤10%, A is a consistent matrix. In section 5, a theorem for consistency of a fuzzy matrix is stated. Since this condition of the theorem is satisfied completely it is expected that the fuzzy pairwise comparisons matrix, $\tilde{A}$ should also be consistent to the same degree as the normal pairwise comparisons matrix is.

USING THE FIRST METHOD TO DETERMINE THE FUZZY WEIGHTS:

- Transform into fuzzy environment using TFNs as depicted in Table 5 where, in this demonstration, the degree of fuzziness is arbitrary setting such as 0.5. The same manner applied in case of trapezoidal fuzzy numbers.
- Constructing the fuzzy pairwise comparisons matrix where, $\tilde{a}_{ji} = (\tilde{a}_{ij})^{-1} = (\gamma_{ij}^{-1}, \beta_{ij}^{-1}, \alpha_{ij}^{-1})$ for all $i,j \in \{1, 2, \ldots, n\}$ as depicted in Table 6.

Table 5: Original fuzzy pairwise comparisons

	f_1	f_2	f_3
f_1	(1, 1, 1)	(1.5, 2, 2.5)	(3.5, 4, 4.5)
f_2		(1, 1, 1)	(0.667, 1.00, 1.5)
f_3			(1, 1, 1)

Table 6: Complete fuzzy pairwise comparisons matrix

	f_1	f_2	f_3
f_1	(1, 1, 1)	(1.5, 2, 2.5)	(3.5, 4, 4.5)
f_2	(0.4, 0.5, 0.667)	(1, 1, 1)	(0.667, 1, 1.5)
f_3	(0.222, 0.25, 0.286)	(0.667, 1, 1.5)	(1, 1, 1)

- Normalized fuzzy pairwise comparisons matrix, $[\tilde{r}_{ij}]$ can be completed using equation (15) where, for each resulting TFN component, α_{ij}, β_{ij}, γ_{ij}>0, $\sum_{i=1}^{3} \alpha_{ij} = 1$, $\sum_{i=1}^{3} \beta_{ij} = 1$, $\sum_{i=1}^{3} \gamma_{ij} = 1$, $\forall i,j \in \{1, 2, 3\}$. And also, $\sum_{i=1}^{3} \delta_{ij} = 1$, in case of trapezoidal fuzzy numbers. The corresponding fuzzy weights can be calculated $\forall i \in \{1, 2, 3\}$ by using the row average approach as shown in Table 7.

Table 7: Normalized fuzzy pairwise comparisons matrix and relative fuzzy weights

	f_1	f_2	f_3	$\sum_{j=1}^{3} \tilde{r}_{ij}$	$\tilde{w}$
f_1	(0.617, 0.571, 0.512)	(0.474, 0.5, 0.5)	(0.677, 0.666, 0.643)	(1.768, 1.737, 1.655)	(0.589, 0.579, 0.552)
f_2	(0.246, 0.286, 0.342)	(0.316, 0.25, 0.2)	(0.129, 0.167, 0.214)	(0.691, 0.703, 0.756)	(0.23, 0.234, 0.252)
f_3	(0.137, 0.143, 0.146)	(0.211, 0.25, 0.3)	(0.194, 0.167, 0.143)	(0.542, 0.56, 0.589)	(0.181, 0.187, 0.196)

- Assuming that U_n and L_n of decision variables provided by the respective DMs are the same where, $x_1, x_2, x_3 \in [0, 1]$. Then formulate the OSP that has a single objective function as follows:

$$\max\ ((0.589, 0.579, 0.552)\ (7x_1 + 3x_2 - 4x_3) + (0.23, 0.234, 0.252)\ x_2 + (0.181, 0.187, 0.196)\ x_3)$$

$$\begin{aligned} \text{st:}\quad & x_1 + x_2 + x_3 \le 3, \\ & x_1 + x_2 - x_3 \le 1, \\ & x_1 + x_2 + x_3 \ge 1, \\ & -x_1 + x_2 + x_3 \le 1, \\ & x_3 \le 0.5, \\ & 0 \le x_1 \le 1, \\ & 0 \le x_2 \le 1, \\ & 0 \le x_3 \le 1. \end{aligned} \tag{19}$$

USING THE SECOND METHOD TO DETERMINE THE FUZZY WEIGHTS:

- In the next step after obtaining Tables 3 (pairwise comparisons matrix) and 4 (normalization matrix), the concept of fuzzy membership function discussed in Section 5.2, can be applied to transform Table 4 into Table 8; that represent a fuzzy pairwise comparisons matrix using fuzzy linguistic variables.

Table 8: Pairwise comparisons matrix using fuzzy linguistic variables.

	f_1	f_2	f_3
f_1	*Equally significant*	*Equally significant*	*Moderately significant*
f_2	*insignificant*	*insignificant*	*Extremely insignificant*
f_3	*Extremely insignificant*	*insignificant*	*Extremely insignificant*

- The fuzzy linguistic variables of the above matrix are then transformed into fuzzy pairwise comparisons matrix with TFNs in Table 9. Table 10 presents the narrow trapezoidal fuzzy numbers case, according to the presented fuzzy scales in Table 2.Other two cases (medium and wide) trapezoidal fuzzy numbers can be also constructed.

Table 9: Fuzzy pairwise comparisons matrix (triangular case).

	f_1	f_2	f_3
f_1	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)
f_2	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.00, 0.10, 0.25)
f_3	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.00, 0.10, 0.25)

Table 10: Fuzzy pairwise comparisons matrix (narrow trapezoidal case).

	f_1	f_2	f_3
f_1	(0.35, 0.45, 0.55, 0.65)	(0.35, 0.45, 0.55, 0.65)	(0.55, 0.65, 0.75, 0.85)
f_2	(0.15, 0.25, 0.35, 0.45)	(0.15, 0.25, 0.35, 0.45)	(0.00, 0.05, 0.15, 0.25)
f_3	(0.00, 0.05, 0.15, 0.25)	(0.15, 0.25, 0.35, 0.45)	(0.00, 0.05, 0.15, 0.25)

- Once the fuzzy pairwise comparisons matrices are completed, the corresponding fuzzy weights can be calculated with TFNs and narrow trapezoidal fuzzy numbers by calculating the average of the elements of each row; the resulting matrices are shown in Tables 11.

Table 11: Relative fuzzy weights.

	Triangular case		Narrow trapezoidal case	
	Row sum	$\tilde{w}$	***Row sum***	$\tilde{w}$
f_1	(1.25, 1.70, 2.15)	(0.417, 0.567, 0.717)	(1.25, 1.55, 1.85, 2.15)	(0.417, 0.517, 0.617, 0.717)
f_2	(0.30, 0.70, 1.15)	(0.100, 0.233, 0.383)	(0.30, 0.55, 0.85, 1.15)	(0.100, 0.183, 0.283, 0.383)
f_3	(0.15, 0.50, 0.95)	(0.050, 0.167, 0.317)	(0.15, 0.35, 0.65, 0.95)	(0.050, 0.117, 0.217, 0.317)

- Formulate OSP; with TFNs using the following single objective function;

$$\max\ ((0.417, 0.567, 0.717)\ (7x_1 + 3x_2 - 4x_3) + (0.1, 0.233, 0.383)\ x_2 + (0.05, 0.167, 0.317)\ x_3), \tag{20}$$

With trapezoidal fuzzy numbers using the following objective function;

$$\max\ ((0.417, 0.517, 0.617, 0.717)\ (7x_1 + 3x_2 - 4x_3) + (0.1, 0.183, 0.283, 0.383)\ x_2 + (0.05, 0.117, 0.217)\ x_3). \tag{21}$$

- ***USING THE GEOMETRIC MEAN METHOD TO DETERMINE THE FUZZY WEIGHTS:***

o Computing the fuzzy weights from the fuzzy pairwise comparisons matrix as follows:

$$\tilde{a}_1 = (\prod_{j=1}^{3} \tilde{a}_{1j})^{1/3} = (\tilde{a}_{11} \times \tilde{a}_{12} \times \tilde{a}_{13})^{1/3} = (5.25, 8, 11.25)^{1/3} = (1.738, 2, 2.241),$$

$$\tilde{a}_2 = (\prod_{j=1}^{3} \tilde{a}_{2j})^{1/3} = (\tilde{a}_{21} \times \tilde{a}_{22} \times \tilde{a}_{23})^{1/3} = (0.267, 0.5, 1.016)^{1/3} = (0.644, 0.794, 1.005),$$

$$\tilde{a}_3 = (\prod_{j=1}^{3} \tilde{a}_{3j})^{1/3} = (\tilde{a}_{31} \times \tilde{a}_{32} \times \tilde{a}_{33})^{1/3} = (0.148, 0.25, 0.429)^{1/3} = (0.529, 0.63, 0.754).$$

$$\tilde{w}_1 = \frac{\tilde{a}_1}{\sum_{i=1}^{3} \tilde{a}_i} = \frac{(1.738, 2, 2.241)}{(2.911, 3.424, 4)} = (0.435, 0.584, 0.77),$$

$$\tilde{w}_2 = \frac{\tilde{a}_2}{\sum_{i=1}^{3} \tilde{a}_i} = \frac{(0.644, 0.794, 1.005)}{(2.911, 3.424, 4)} = (0.161, 0.232, 0.345),$$

$$\tilde{w}_3 = \frac{\tilde{a}_3}{\sum_{i=1}^{3} \tilde{a}_i} = \frac{(0.529, 0.63, 0.754)}{(2.911, 3.424, 4)} = (0.132, 0.184, 0.259).$$

o Formulate the MLP problem model using the following single objective function:

$$\max\ ((0.435, 0.584, 0.77)\ (7x_1 + 3x_2 - 4x_3) + (0.161, 0.232, 0.345)\ x_2 + (0.132, 0.184, 0.259)\ x_3). \tag{22}$$

Table 12 shows the different relative fuzzy weights using FAHP and AHP approaches. By solving problems (19), (20), (22), get out the optimal solution for MLP problem (1) with TFNs where the results shown in Table 13 represent the achieved satisfactory solutions. The satisfactory solution is; x_1 = (1.0, 1.0, 1.0), x_2, x_3 = (0.5, 0.5, 0.5) and f_1= (6.5, 6.5, 6.5), f_2, f_3 = (0.5, 0.5, 0.5). The procedure steps can be applied using Table 2 and problem (21) in trapezoidal fuzzy numbers case.

Table 12: Relative fuzzy weights using FAHP and AHP.

	Relative weights			
	FAHP approach (TFNs case)			AHP approach
	1[st] method	2[nd] method	3[rd] method	
w_1	(0.589, 0.579, 0.552)	(0.417, 0.567, 0.717)	(0.435,0.584, 0.77)	0.58
w_2	(0.23, 0.234, 0.252)	(0.100, 0.233, 0.383)	(0.161,0.232, 0.345)	0.24
w_3	(0.181, 0.187, 0.196)	(0.050, 0.167, 0.317)	(0.132,0.184, 0.259)	0.18

7. DISCUSSION AND CONCLUSION

Hierarchical decision making often involves many uncertain factors and its situation can be modeled as MLP problem which can be defined as p-person, non-zero sum game with perfect information where each player moves sequentially from top to bottom. Since these situations look for the satisfaction among DMs, AHP approaches can be used to give the relative weights of their objective functions where these weights express the preferences/importance or significance of DMs and based on the question “which objective has more significance and by how much?”. While dealing with vagueness of human thought and uncertainty of future events, and in order to take the imprecision of human qualitative assessments into consideration, FAHP is used for the estimation of the significance of MLP objectives. To achieve the satisfactory solution of the MLP problem, a single objective function can be formed while converting the hierarchical system into SOP. Fuzzification means using fuzzy numbers instead of the exact or crisp numbers. A major contribution of fuzzy set theory is its capability of representing vague data. In FAHP, pairwise comparisons are done using fuzzy linguistic preference variables. In this article, the FAHP is used for achieving the relative weights of MLP problem's objectives throughout three different methods by eigen value and geometric mean. Then, a compromise weighted approach is applied to solve MLP problem which can be applicable as an approximation solution for; ML fractional programming problems and nonlinear MLP problems. The presented approach is effective tool for finding a satisfactory solution with suitable limitation of the decision variables with upper and lower bounds. Three presented methods can be applied with TFNs or trapezoidal fuzzy numbers which includes many shapes such as; narrow, medium and wide shapes. The weighting approach for solving MLP problems that represent the hierarchical structure situations is advantage with its flexibility; one in the allowed limitation of the preferences of DMs for decision variables they control and second issue, the given arbitrary fuzzifying values while constructing the fuzzy numbers. These factors make the solution approach be a good opportunity to discuss the uncertainty factors that usually counter DMs in many of the hierarchical decision making situations. The resulted relative weights shown in Table 12, and the satisfactory solutions mean that the presented approach through the different methods can produces results which are very close or improved to the results obtained by classical AHP approach. But some relative fuzzy weights by geometric mean are the nearest of the crisp weights by AHP.

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