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COMMON FIXED POINT THEOREM USING "E.A LIKE" PROPERTY AND IMPLICIT RELATION IN INTUITIONISTIC METRIC SPACES

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ABSTRACT

The present paper deals with "E.A. Like property and its application in proving common fixed point result in a intitionistic fuzzy metric space.

Keywords: *Weakly compatible mappings; E.A property; implicit relation, E.A. like property.*

AMS Subject Classification: *47H10, 54H25.*

1. INTRODUCTION

Atanassov [9] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [10] and later there has been much progress in the study of intuitionistic fuzzy sets [3,7]. In 2004, Park [8] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [1]. Fixed point theory has important applications in diverse.

In 1986, Jungck [6] introduced the notion of compatible maps for a pair of self mappings. Aamri and El Moutawakil [11] generalized the concept of non-compatibility by defining the notion of property (E.A) and in 2005, Liu, Wu and Li [19] defined common (E.A) property in metric spaces and proved common fixed point theorems under strict contractive conditions. Jungck and Rhoades [5] initiated the study of weakly compatible maps in metric space and showed that every pair of compatible maps is weakly compatible but reverse is not true. Fixed point theory has important applications in diverse disciplines of mathematics, statistics, engineering, and economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc. Recently, Kumar [17] established some common fixed point theorems in intuitionistic fuzzy metricspace using property (E.A). We generalize his result[18] for E.A. Like property, and prove fixed theorems for weakly compatible mappings via an implicit relation in intuitionistic fuzzy metric spaces.

2. PRELIMINARIES

Definition 2.1: A binary operation $*$: [0; 1] x [0; 1] $\rightarrow$ [0; 1] is continuous t-norm if satisfies the following conditions:
(1) $*$is commutative and associative,
(2) $*$is continuous,
(3) $a * 1 = a$ for all $a \in [0; 1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0; 1]$.

Definition 2.2: A binary operation $\lozenge$: [0; 1]x[0; 1] $\rightarrow$ [0; 1] is continuous t-conorm if satisfies the following conditions:
(1) $\lozenge$ is commutative and associative,
(2) $\lozenge$ is continuous,
(3) $a \lozenge 0 = a$ for all $a \in [0; 1]$,
(4) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0; 1]$.

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Alaca *et al*. [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzymetric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzymetric space due to Kramosil and Michalek [ ] as:

Definition 2.3: A 5-tuple (X; M; N; *; $\Diamond$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2x[0;\infty)$ satisfying the following conditions:
(1) $M(x, y, t) + N(x,y, t) \le 1$ for all $x, y \in X$ and $t > 0$,
(2) $M(x, y, 0) = 0$ for all $x, y \in X$,
(3) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$,
(4) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$,
(5) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$,
(6) for all $x, y \in X$, $M(x, y,.) : [0, \infty) \to [0, 1]$ is left continuous,
(7) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$,
(8) $N(x, y, 0) = 1$ for all $x, y \in X$,
(9) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$,
(10) $N(x, y, t) = N(y, x, t)$ for all $x; y \in X$ and $t > 0$,
(11) $N(x, y, t) \Diamond N(y, z, s) = N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$,
(12) for all $x, y \in X$, $N(x, y, .) : [0, \infty) \to [0, 1]$ is right continuous,
(13) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4: In intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$), M(x, y, *) is non-decreasing and N(x, y, $\Diamond$) is non-increasing for all $x, y \in X$.

Definition 2.5: Let (X, M, N,*, $\Diamond$), be an intuitionistic fuzzy metric space. Then: a sequence $\{x_n\}$in X is said to be Cauchy sequence if, for all t > 0 and p > 0,
$\lim_{n\to\infty} M(x_{n+p}, x, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x, t) = 0$:

Definition 2.6: a sequence $\{x_n\}$in X is said to be convergent to a point $x \in X$ if, for all t > 0 and p > 0,
$\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$:) a sequence fxng in X is said to be convergent to a point x 2 X if, for all t > 0,

Definition 2.7: An intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$), is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.8: A pair of self mappings (T, S) of an intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$), is said to satisfy the property (E.A) if there exist a sequence $\{x_n\}$in X such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = z$ in X.

Definition 2.9: A pair of self mappings (T, S) of an intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$), is said to satisfy the property (E.A) like property if there exist a sequence $\{x_n\}$in X such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in T(X)$ or $z \in T(X)$ i.e $z \in T(X) \cap S(X)$

Definition 2.10: Two pairs (A,S) and (B,T) of self mappings of an intuitionistic fuzzy metric space (X,M,N,*, $\Diamond$), are said to satisfy the common (E.A) property if there exist two sequences $\{x_n\}$ and$\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z \text{ in X for some z in X.}$$

Definition 2.11: Two pairs (A, S) and (B, T) of self mappings of an intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$), are said to satisfy the common (E.A) like property if there exist two sequences $\{x_n\}$ and$\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z \text{ where } z \in T(X) \cap S(X) \text{ or } z \in A(X) \cap B(X)$$

Example 2.12: Consider (X, M, N,*, $\Diamond$), be an intuitionistic fuzzy metric space with M(x, y, t)= $\dfrac{t}{t+d(x,y)}$ and

N(x, y, t)= $\dfrac{d(x,y)}{t+d(x,y)}$

Define self mappings A, B, S and T on X as Ax = x/3, Bx = -x=/3, Sx = x, and Tx = -x for all x $\in$ X. Then with sequences $\{x_n\} = \{1/n\}$ and $\{y_n\} = \{-1/n\}$ in X, one can easily verify that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 0$

Therefore, pairs (A, S) and (B, T) satisfies the common E.A. property.

Example 2.13: Consider (X, M, N,*, $\Diamond$), be an intuitionistic fuzzy metric space with M(x, y, t) = $\dfrac{t}{t+d(x,y)}$ and

N(x, y, t)= $\dfrac{d(x,y)}{t+d(x,y)}$

Define self mappings A, B, S and T on X as

Ax = $\begin{cases} 1, & x \le 1 \\ 2-x, & x \ge 1, \end{cases}$ Sx = $\begin{cases} 2x-4, & x \ge 2 \\ 2-x, & x \le 2 \end{cases}$

Tx = $\begin{cases} x, & x \le 1 \\ 1, & x \ge 1, \end{cases}$ Bx= $\begin{cases} 1, & x \le 1 \\ \dfrac{1}{x}, & x \ge 1 \end{cases}$

for all x $\in$ X. Then with sequences $\{x_n\} = \{1+1/n\}$ and $\{y_n\} = \{1-1/n\}$ in X, one can easily verify that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 1$ and A(X)=$(-\infty,1]$, B(X)=(0,1], T(X)= $(-\infty,1]$, S(X)=X.

Therefore, pairs (A, S) and (B, T) satisfies the common E.A. property

Definition 2.14: A pair of self mappings (T,S) of an intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$),is said to be weakly compatible if they commute at coincidence points, i.e., if Tu = Su for some u $\in$ X, then TSu = STu.

Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces using Common (E.A like) Property and Implicit Relation

3. MAIN RESULT

Implicit relations play important role in establishing of common fixed point results.

Let M_6 be the set of all continuous functions $\varphi : [0,1]^6 \to R$ and $\psi : [0,1]^6 \to R$ satisfying the following conditions:

(A) φ(u, 1, u, 1, 1, u) < 0 for all u $\in$ (0, 1),

(B) φ (u, 1, 1, u, u, 1) < 0 for all u $\in$ (0, 1),

(C) φ (u, u, 1, 1, u, u) < 0 for all u $\in$ (0, 1),

(D) ψ (v, 0, v, 0,0, v) > 0 for all v$\in$ (0, 1),

(E) ψ (v, 0, 0, v, v, 0) > 0 for all v $\in$ (0, 1),

(F) ψ (v, v, 0, 0, v, v) > 0 for all v $\in$ (0, 1).

In 2012, S. manro, S. Kumar, Sanjay Kumar, S. S. Bhatia proved following result

Theorem3.1: Let A, B, S and T be self mappings of an intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$), satisfying the following:

(1) the pair (A, S) and (B, T) satisfies the E.A. property,

(2) for any x, y $\in$ X, φ and ψ in M^6 and for all t > 0,

$$\varphi\ (M(Ax, By, t), M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t)) \ge 0$$

$$\psi\ (N(Ax, By, t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Sx, By, t), N(Ty, Ax, t)) \le 0$$

(3) $A(X) \subseteq T(X)$ or $B(X) \subseteq S(X)$.
(4) the pairs (A, S) and (B, T) are weakly compatible.
(5) S(X) and T(X) are closed subspace of X.

Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed.

Now we prove the following:

Theorem3.2: Let A, B, S and T be self mappings of an intuitionistic fuzzy metric space $(X, M, N,*, \Diamond)$, satisfying the following:
(1) the pair (A, S) and (B, T) satisfies the E.A. like property,
(2) for any $x, y \in X$, φ and ψ in M^6 and for all $t > 0$,

$$\varphi\,(M(Ax, By, t), M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t)) \geq 0$$
$$\psi\,(N(Ax, By, t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Sx, By, t), N(Ty, Ax, t)) \leq 0$$

(3) $A(X) \subseteq T(X)$ or $B(X) \subseteq S(X)$.
(4) the pairs (A, S) and (B, T) are weakly compatible.
(5) S(X) and T(X) are closed subspace of X.

Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed.

Proof: Since (A, S) and B, T) satisfies the E.A. like property therefore there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z \text{ where } z \in T(X) \cap S(X) \text{ or } z \in A(X) \cap B(X)$$

Suppose $z \in T(X) \cap S(X)$, we have

$\lim_{n\to\infty} Ax_n = z \in S(X)$ then $z=S(u)$ for some $u \in X$.

Put $x=u$ and $y=y_n$ in (2)

$$\varphi\,(M(Au, By_n, t), M(Su, Ty_n, t), M(Su, Au, t), M(Ty_n, By_n, t), M(Su, By_n, t), M(Ty_n, Au, t)) \geq 0$$

$$\psi\,(N(Au, By_n, t), N(Su, Ty_n, t), N(Su, Au, t), N(Ty_n, By_n, t), N(Su, By_n, t), N(Ty_n, Au, t)) \leq 0$$

$\Rightarrow$ As $n \to \infty$, we have

$$\varphi\,(M(Au, z, t), M(Su, z, t), M(Su, Au, t), M(z, z, t), M(Su, z, t), M(z, Au, t)) \geq 0$$

$$\psi\,(N(Au, z, t), N(Su, z, t), N(Su, Au, t), N(z, z, t), N(Su, z, t), N(z, Au, t)) \leq 0$$

And $Su=z$

$$\varphi\,(M(Au, z, t), M(z, z, t), M(z, Au, t), M(z, z, t), M(z, z, t), M(z, Au, t)) \geq 0$$

$$\psi\,(N(Au, z, t), N(z, z, t), N(z, Au, t), N(z, z, t), N(z, z, t), N(z, Au, t)) \leq 0$$

$$\Rightarrow \varphi\,(M(Au, z, t), 1, M(z, Au, t), 1, 1, M(z, Au, t)) \geq 0$$

$$\psi\,(N(Au, z, t), 0, N(z, Au, t), 0, 0, N(z, Au, t)) \leq 0$$

Therefore $Au=z=Su$. Which shows that u is coincidence point of the pair (A, S). The weak compatibility of A and S implies that $ASu=SAu$ and then

$$AAu=ASu=SAu=SSu. \qquad (*)$$

Since T(X) is also a closed subset of X, therefore $\lim_{n\to\infty} Ty_n = z$ in T(X) and hence there exists v $\in$ X such that Tv = z = Au = Su. Now, we show that Bv = z.

Take x = u, y = v, we have

φ (M(Au, Bv, t), M(Su, Tv, t), M(Su, Au, t), M(Tv, Bv, t), M(Su, Bv, t), M(Tv, Au, t)) ≥ 0

ψ (N(Au, Bv, t), N(Su, Tv, t), N(Su, Au, t), N(Tv, Bv, t), N(Su, Bv, t), N(Tv, Au, t)) ≤ 0

$\Rightarrow \varphi$ (M(z, Bv, t), M(z, z, t), M(z, z, t), M(z, Bv, t), M(z, Bv, t), M(z, z, t)) ≥ 0

ψ (N(z, Bv, t), N(z, z, t), N(z, z, t), N(z, Bv, t), N(z, Bv, t), N(z, z, t)) ≤ 0

$\Rightarrow \varphi$ (M(z, Bv, t),1,1, M(z, Bv, t), M(z, Bv, t), 1) ≥ 0

ψ (N(z, Bv, t), 0, 0, N(z, Bv, t), N(z, Bv, t),0) ≤ 0

Therefore, Bv = z = Tv which shows that v is a coincidence point of the pair (B; T).

The weak compatibility of B and T implies that BTv=TBv and then

BBv=BTv=TBv=TTv (**)

From (*) and (**), we have

Az=ASu=SAu=Sz,

Bz=BTv=TBv=Tz (***)

Take x = z, y = v, we have

φ (M(Az, Bv, t),M(Sz, Tv, t), M(Sz, Az, t), M(Tv, Bv, t), M(Sz, Bv, t), M(Tv, Az, t)) ≥ 0

ψ (N(Az, Bv, t), N(Sz, Tv, t), N(Sz, Az, t), N(Tv, Bv, t), N(Sz, Bv, t), N(Tv, Az, t)) ≤ 0

$\Rightarrow \varphi$ (M(Az, z, t), M(Az, z, t), M(Az, Az, t),M(z, z, t), M(Az, z, t), M(z, Az, t)) ≥ 0
ψ (N(Az, z, t), N(Az, z, t), N(Az, Az, t), N(z, z, t), N(Az, z, t), N(z, Az, t)) ≤ 0

$\Rightarrow \varphi$ (M(Az, z, t), M(Az, z, t), 1, 1, M(Az, z, t), M(z, Az, t)) ≥ 0
ψ (N(Az, z, t), N(Az, z, t), 0, 0, N(Az, z, t), N(z, Az, t)) ≤ 0

Therefore, Az=z=Sz.

Similarly, Bz = Tz = z. Hence, Az = Bz = Sz = Tz=z, and z is common fixed point of A, B, S and T.

Uniqueness: Let z and w be two common fixed points of A, B, S and T. Take x=z and y=w in (2)

φ (M(Az, Bw, t), M(Sz,Tw, t), M(Sz, Az, t), M(Tw, Bw, t), M(Sz, Bw,t), M(Tw, Az, t)) ≥ 0

ψ (N(Az, Bw, t), N(Sz,Tw, t), N(Sz, Az, t), N(Tw, Bw, t), N(Sz, Bw, t), N(Tw, Az, t)) ≤ 0

$\Rightarrow \varphi$ (M(z, w, t), M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t), M(w, z, t)) ≥ 0

ψ (N(z, w, t), N(z, w, t), N(z, z, t), N(w, w, t), N(z, w, t), N(w, z, t)) ≤ 0

$\Rightarrow \varphi$ (M(z, w, t), M(z, w, t), 1, 1, M(z, w, t), M(w, z, t)) ≥ 0

ψ (N(z, w, t), N(z, w, t), 0, 0, N(z, w, t), N(w, z, t)) ≤ 0

$\Rightarrow$ z=w. hence there is unique fixed point.

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